# Concurrent Collective Strategy Diffusion of Multiagents: The Spatial Model and Case Study

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***Abstract***

In Section IV, we make a case study of queue orientation to test the proposed formal model. In Section V, we discuss the related work. Finally, we present our conclusion and future work in Section VI.

## II. Collective Strategy Diffusion Phenomenon With Concurrent Form in Multiagent Societies

### A. Collective Strategy Diffusion Phenomenon

Diffusion phenomenon is the penetration and imitation of strategies among agents. We initiated a study on how to evolve individual social strategies to global social law through hierarchical agent diffusion [18]. In our original model, each agent has its own social strategy at the first stage of the system. Further, we assume that the agents possess "social ranks" and the ranks are known by all agents. As the system runs, the strategies of superior agents will tend to diffuse to (be adopted by) the junior agents. Given the right conditions and enough time, a global social law may be finally established. A simple example is given as follows:

> Two businessmen enter the same conference room. One starts to smoke but the other complains. Upon learning that the smoker has a higher position in the same company, the complainer changes his strategy of "smoking is not permitted" to "go ahead." This means that the social law in the conference room has become "smoking is permitted." [Scenario 1: diffusion by rank]

Only the diffusion of the social strategies of superior agents to junior agents was considered in [18]. However, such a situation is not truly representative of reality. In real society, there are so many collective intentions and practices [19], [20], and the social strategies shared by many junior agents can also influence the superior agents. The following example demonstrates this:

> You like smoking very much and you occupy the highest position in your company. However, if all other members dislike smoking, you will probably stop smoking, i.e. your social strategy has been trumped by that of the majority. Therefore, the social law of your office is now "no smoking." [Scenario 2: diffusion by numbers]

The interplay between rank, strength of strategy support, and the numbers of supporters is extremely complex. The emergence of a social law may not prevent some agents from maintaining their original strategies or indeed adopting contra strategies. Ct diffusion process, we also need to consider the impacts of other diffusion processes that take place concurrently.

## III. Proposed Model

### A. General Model for Collective Diffusion

*1) Authority of Social Strategy:* The agent with higher authority has higher dominance to diffuse. If a social strategy is accepted by many agents, its authority will be strong. However, different agents will contribute different amounts to the authority of a social strategy. A superior agent strengthens a strategy's authority far more than a junior agent.

*Definition 1:* Social position of agent $i$ can be a function $p_i \to [0, \vartheta]$, where $\vartheta$ is a natural number. $p_i > p_j \Rightarrow$ agent $i$ has superior position to agent $j$.

In this paper, our basic idea for the authority of social strategy is that if agent $a$ has a social strategy $s$, $a$ is implicitly conferring some importance to the authority of $s$. Then, how much importance does an agent $a$ confer to its social strategy $s$? Let $p_a$ represent the social position of $a$, then agent $a$ confers $p_a$ units of rank to $s$.

*2) Overlay Group of Social Strategy:* The agents that share a social strategy are called the *overlay group* of such strategy. Let $G(s)$ represent the overlay group of social strategy $s$, we have

$$\forall_s, \qquad G(s) = \{u | \text{agent } u \text{ accepts the social strategy } s\}. \tag{1}$$

Let $G(s)$ be the overlay group of social strategy $s$, the rank of $s$ can be defined as follows:

$$\forall_s, \qquad \text{Rank}(s) = \sum_{u \in G_s} p_u. \tag{2}$$

Obviously, the rank of a social strategy is determined by the number and social positions of its overlay group members. The diffusion strength of social strategy $s$ is in direct proportion to its rank. Moreover, we will use the concept of group rank to represent the social strategy rank, i.e.,

$$\forall_s, \qquad \text{Rank}(G(s)) = \text{Rank}(s).$$

*3) Spatial Distance Between Agent and Overlay Group:* In the related benchmark research on collective motion of multiagents [1]–[7], the authors always let all agents locate in a euclidian space. Therefore, in this paper, we also consider the spatial distance for simplifying the explanation of model.

It is very simple to compute the spatial distance between two agents, but not the one between an agent and a group. As stated before, each agent in the group strengthens the authority of strategy to a different extent. Therefore, we introduce the factor of social position into the definition of spatial distance between an agent and a group as follows.

*Definition 2:* If $d(a,u)$ denotes the spatial distance between agent $a$ and agent $u$, $|G|$ denotes the number of agents in group $G$, and $p_u$ denotes the social position of agent $u$, then the spatial distance between agent $a$

let the social strategy of agent $a$ be $s_1$ and the social strategy of overlay group $G$ be $s_2$, we define the counteracting force from agent $a$ to overlay group $G$ as

$$\mathrm{CF}_{a\to G} = g\left(\alpha_1 \cdot DL_{s_1 s_2} \times \alpha_2 \cdot \left(\frac{p_a}{1/|G| \times \sum_{u\in G} p_u}\right)\right) \quad (5)$$

where $g$ is a monotone increasing function, and $\alpha_1$ and $\alpha_2$ are two weighting parameters. Also, we can directly use

$$\alpha_1 \cdot DL_{l_1 l_2} \times \alpha_2 \cdot \left(p_a \Big/ \left(\frac{1}{|G|}\sum_{u\in G} p_u\right)\right)$$

to denote the counteracting force.

From (5), we can see that the counteracting force of $a$ will become very strong if we set the position of $a$ as a very high value. Therefore, now the social strategy of $a$ cannot be changed by other agents, which is the explanation of scenario 3.

*7) Collective Diffusion Criterion of Social Strategy:* The collective diffusion criterion of agent social strategy needs to consider both $\mathrm{IF}_{G\to a}$ and $\mathrm{CF}_{a\to G}$. The more $\mathrm{IF}_{G\to a}$ is and the less $\mathrm{CF}_{a\to G}$ is, the more likely it is that agent $a$ will adopt or move toward the social strategy of group $G$. Therefore, we use the ratio of $\mathrm{IF}_{G\to a}$ to $\mathrm{CF}_{a\to G}$ as the diffusion criterion. If the ratio exceeds a predefined value, the social strategy of agent $a$ will change by some amounts.

We can predefine two parameters, $\xi$ and $\eta$, according to the actual situation being simulated. If the value of $\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}$ is more than $\xi$, the social strategy of $a$ will completely switch to that of $G$. If the value of $\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}$ is less than $\xi$ but more than $\eta$, then the social strategy of $a$ will change to some extent but not equal to that of $G$. While the value of $\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}$ is less than $\eta$, the social strategy of $a$ remains unchanged

$$s'_a = \begin{cases} s_G, & \text{if } \xi \le \mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G} \\ \mathbb{C}(s_a, s_G), & \text{if } \eta \le \mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G} < \xi \\ s_a, & \text{if } \mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G} < \eta \end{cases} \quad (6)$$

where $s'_a$ is the new social strategy of $a$ after one step in the simulation, and $\mathbb{C}(s_a, s_G)$ is the coordination between $s_G$ and $s_a$.

*8) Conflict Between the Diffusions of Different Overlay Groups:* If there are two overlay groups $G_1$ and $G_2$, and $\mathrm{IF}_{G_1\to a}/\mathrm{CF}_{a\to G_1}$ is the same as $\mathrm{IF}_{G_2\to a}/\mathrm{CF}_{a\to G_2}$, we can say that there is a conflict between the diffusions of two overlay groups. In such a situation, agent $a$ will select to incline to the social strategy that is closer. Therefore, if $\mathrm{IF}_{G_1\to a}/\mathrm{CF}_{a\to G_1} == \mathrm{IF}_{G_2\to a}/\mathrm{CF}_{a\to G_2}$, $s_1$ is the social strategy of $G_1$, $s_2$ is the social strategy of $G_2$, and $\to$ denotes the "incline" process described in Section III-A7, then the new social strategy of $a$ will be changed as follows: $s'_a \to s_1$ if $D_{s_1 s_a} \le D_{s_2 s_a}$ and $s'_a \to s_2$ if $D_{s_1 s_a} > D_{s_2 s_a}$.

*9) Goal Function of Collective Diffusion:* In this paper, we mainly consider the collective diffusion based on the authorities of strategies; thus, we want the strategies with higher authorities to be easily accepted by other agents. Therefore, we define the performance criterion of diffusion as a measurement of the winnowing process of social strategies. Let $N_S$ be the number of social strategies in the system after diffusion, $|G(s)|$ be the number of agents in the overlay group of social strategy $s$, then the performance criterion of the diffusion can be defined as

$$P_S = \frac{1}{N_S}\sum_{s\in S}(\mathrm{Rank}(s)\cdot|G(s)|). \quad (7)$$

Higher value of $P_S$ indicates that fewer dominant social strategies have survived and also shows that better diffusion performance can be obtained.

*10) Sequential Collective Diffusion Process:* In the sequential diffusion process, the social strategy with the strongest authority will first diffuse to other agents that belong to the other social strategies' overlay groups. After that, the social strategy with the second strongest authority will diffuse to other agents that belong to the agents with more junior authority social strategies, until there are no diffusions to take place.

Let $A$ be the set of agents in the system and $s_a$ be the social strategy of agent $a$, the diffusion process can be shown as Algorithm 1.

**Algorithm 1**

*Input S*; /*the set of initial social strategies of multiagents*/
*Input SL*; /*the whole set of social strategy available to agents*/
***Do***
[redacted]
***While*** $SL'$ is not empty:
{Select the social strategy with the strongest authority from $SL'$: $s^*$;
***Do***
{***For*** $\forall a \in A' - G(s^*)$:
***If*** $\eta \le IF_{G(s^*)\to a}/CF_{a\to G(s^*)}$
{ $G(s_a) = G(s_a) - \{a\}$;
$Rank(s_a) = Rank(s_a) - p_a$
***If*** $G(s_a) = \{\}$ ***then*** $SL = SL - \{s_a\}$;
Compute $s'_a$ according to Equation (6);
$G(s'_a) = G(s'_a) \cup \{a\}$;
$Rank(s'_a) = Rank(s'_a) + p_a$;}
} ***Until*** the $P_S$ does not change any more;
$SL' = SL' - \{s^*\}$; $A' = A' - G(s^*)$;}
} ***Until*** the $P_S$ does not change any more;

## B. Dealing With the Concurrent Form and Unification Trend

In the previous section, we only consider the sequential form of collective diffusion. However, such a situation is simple and sometimes may not be real in the society. When diffusion processes take place concurrently with the unification trend, each agent will incline to select the social strategy with strong impact strength as well as the social strategy that will be accepted by other agents in the system. Therefore, when we decide the change probability of an agent's social strategy, we will consider the *social strategy impact strength* and the *average social strategy* of all agents together.

In the diffusion, an agent will go toward a new social strategy that can be called *inclined social strategy*. Next, we will consider the impact forces of different inclined social strategies to an agent by dealing with the concurrent mechanism and unification trend in the diffusion.

*1) Trend to Strong Social Strategy:* In (2), we only simply define the impact strength of a social strategy as its authority. Now, for considering all diffusion processes concurrently, we think that the impact strength of a social strategy will be decided by the collective positions and geographical distribution of the agents within its overlay group.

*Definition 3:* Let $s_s(x, y)$ denote the impact strength of social strategy $s$ at the place of $(x, y)$ and $i\Theta(x, y)$ denote that agent $i$ does not locate at the place of $(x, y)$, then $s_s(x, y)$ is defined as

$$s_s(x, y) = \sum_{i \in G(s), i\Theta(x,y)} \frac{\gamma_1 p_i}{\gamma_2 \cdot \sqrt{(x_i - x)^2 + (y_i - y)^2}} \quad (8)$$

where $\gamma_1$ and $\gamma_2$ are parameters to determine the relative importance of the two factors. If agent $a$ locates at the place of $(x, y)$ and the set of social strategies in the system is $S$, then the probability that $a$ inclines to select social strategy $s$ in diffusion can be initially defined as

$$\text{prob}_i^s = \frac{s_s(x, y)}{\sum_{m \in S} s_m(x, y)}. \quad (9)$$

Obviously, this definition for change probability in diffusion is simple without considering other concurrent diffusion processes and unification trend.

*2) Concurrent Trend to Current Average Social Strategy:* Now, the agent will incline to select not only the social strategy with strong impact strength, but also the average social strategy that can satisfy the trend to unification.

Then, what is the average social strategy? Obviously, it is not the simple average value of all social strategies. In this paper, we think that the average social strategy is the average of social strategies by regarding their ranks.

*Definition 4:* Let $S$ be the set of social strategies in the system and $s$ is a social strategy in $S$, the *current average social strategy of the system* (CASS) can be defined as

$$\text{CASS} = \sum_{s \in S} \frac{\text{Rank}(s)}{\sum_{m \in S} \text{Rank}(m)} \times s. \quad (10)$$

The inclination of an agent will compromise the two factors of *social strategy impact strength* and *CASS* together. Now we define the change probability by modifying (9) to

$$\begin{aligned}\text{prob}_i^s * &= \frac{\beta_1 \text{prob}_i^s + \beta_2 (1 - DL_{s,\text{CASS}}/\Im)}{\sum_{n \in S} (\beta_1 \cdot \text{prob}_i^n + \beta_2 \cdot (1 - DL_{n,\text{CASS}}/\Im))} \\ &= \frac{\beta_1 \cdot (s_s(x, y)/\sum_{m \in S} s_m(x, y)) + \beta_2 \cdot (1 - DL_{s,\text{CASS}}/\Im)}{\sum_{n \in S} (\beta_1 \cdot (s_n(x, y)/\sum_{m \in S} s_m(x, y)) + \beta_2 \cdot (1 - DL_{n,\text{CASS}}/\Im))}\end{aligned} \quad (11)$$

where $\beta_1$ and $\beta_2$ are parameters to determine the relative importance of the two factors, and $\Im$ is the maximum difference of strategies in the system. Therefore, the probability that agent $a$ selects social strategy $s$ is determined by not only the impact strength of $s$ but also the difference between $s$ and CASS.

*3) Concurrent Trend to Expected Average Social Strategy:* *Definition 5:* Let the current social strategy of agent $i$ be $s_i$ and its locality be $(x_i, y_i)$, $S$ be the set of social strategies in the system, and $s$ be a social strategy in $S$, then the expected social strategy (ESS) of agent $i$ after the diffusion can be defined as

$$\text{ESS}_i = \sum_{s \in S} \text{prob}_i^s * \times s. \quad (12)$$

*Definition 6:* Let $n$ be the number of agents in the system, $S$ be the set of social strategies in the system, and $s$ be a social strategy in $S$, then the expected average social strategy of the whole system can be defined as

$$\text{EASS} = \sum_{i=1}^{n} \left( \frac{p_i}{\sum_i p_i} \text{ESS}_i \right) = \sum_{i=1}^{n} \left( \frac{p_i}{\sum_i p_i} \sum_{s \in S} \text{prob}_i^s * \times s \right). \quad (13)$$

As said before, the agent has three inclinations: the social strategy with strong impact strength, the CASS, and the EASS. Therefore, we need to compromise these three inclinations together.

Then, the real probability that agent $i$ goes toward social strategy $s$ given in (14), shown at the bottom of the page, where $\lambda_1$, $\lambda_2$, and $\lambda_3$ are parameters to determine the relative importance of the three factors. Obviously, $\sum_s \text{prob}_i^s ** = 1$.

*4) Performance Judgment of the Diffusion:* In (7), we define the original goal function of diffusion. Now, for the trend of unification, we will try to make the difference between all agents' strategies and the CASS after diffusion is minimized. Let the number of agents be $n$, then the total difference between all agents' strategies and the CASS can be called as *variation index* (VI)

$$\text{VI} = \frac{1}{}$$

Obviously, the less the VI is, the higher the unification trend result can be received by diffusion.

Therefore, we can modify (7) to

$$P'_S = \frac{P_S}{\text{VI}} = \begin{cases} \dfrac{n \cdot \sum_{s\in S} (\text{Rank}(s) \cdot |G(s)|)}{N_S \cdot \sum_{i=1}^{n} DL_{s_i \text{CASS}}}, & \text{if } \sum_{i=1}^{n} DL_{s_i \text{CASS}} \neq 0 \\ \dfrac{n \cdot \sum_{s\in S} (\text{Rank}(s) \cdot |G(s)|)}{N_S}, & \text{else.} \end{cases} \tag{16}$$

*5) Saturation of the Diffusion:* Then, how can we fix it on the end of the diffusion? The end of diffusion can be called as *diffusion saturation*. In the status of diffusion saturation, the diffusion cannot proceed any more. Thus, the saturation of the diffusion can be defined as

$$\text{EASS} == \text{CASS}. \tag{17}$$

*6) Concurrent Collective Diffusion Process:*

**Algorithm 2**

*Input A, S; /*the agent system, and the initial set of social strategies of multiagents*/*
*Input SL; /*the whole set of social strategy space available to agents*/*
***Do***
*{**For** each agent $i \in A$ :*
*{**For** each social strategy $s \in SL$ :*
*Compute $prob_i^s$ ** ;*
*Select the social strategy with the highest $prob_i^s$ ** , denoted as $s^{\circledast}$ ;*
*The social strategy of agent i inclines to $s^{\circledast}$ according to Equation (6);}*
*} **until** EASS == CASS ;*
*Output S and $P_S'$ .*

## IV. Case Study

### A. Introduction for the Case of Queue Orientation

To explain our model, we use the case of an agent system that reproduces a crowd of strangers standing on a soccer field (modeled by a 2-D space). At the initial stage, the orientation of each agent (its strategy) is quite random. What we are looking for is the emergence of a unique strategy: most agents face the same direction. Therefore, at the initial stage of our agent system, each agent can stand with its orientation; after continuous diffusion, the agents may stand with a unique orientation. Obviously, if many agents or some superior agents stand for a unique orientation, then the other many individuals had no choice but to fall back on such strong orientation.

We can let agent stand with one of the eight orientations in Fig. 2(a). Therefore, the social strategy of an agent is its standing orientation. Let $n$ be the number of agents, then we can use an array to denote the social strategies of agents. $s_i \to \{1, \ldots, 8\}, 1 \le i \le n$, represents social strategy of agent $i$.

Therefore, the multiagent system in our case can be formally defined as follows.

*Definition 7:* The multiagent system simulating people standing orientations is a tuple $\langle G, A, \lambda \rangle$, which consists of a 2-D grid $G$ with the set of places ($x$, $y$), a set $A$ of agents, and a placing function $\lambda : (x, y) \to \{1, 2, \ldots, 8\}$. If $1 \le m \le 8$, then "$\lambda(x, y) = m$" denotes that there is an agent at the place of ($x$, $y$) whose social strategy is $m$. If $m = 0$, then "$\lambda(x, y) = m$" denotes that there are no agents at the place of ($x$, $y$).

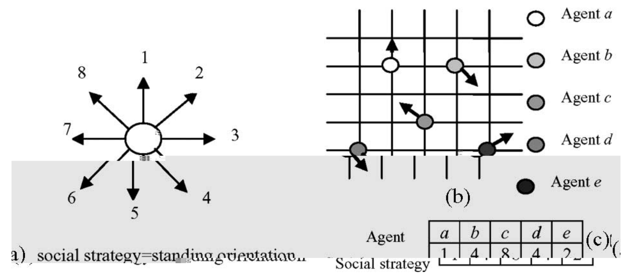


Fig. 2. Case of people standing orientation.

| Agent | *a* | *b* | *c* | *d* | *e* |
|---|---|---|---|---|---|
| Position | 2 | 3 | 3 | 3 | 4 |

(a)

| Social Strategy | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Overlay agents | *a* | *e* | \ | *bd* | \ | \ | \ | *c* |
| Authority | 1 | 4 | 0 | 6 | 0 | 0 | 0 | 3 |

(b)

Fig. 3. Social positions, strategies, and authorities of the agent system in Fig. 2(b).

Social strategy / Social strategy

| / | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** |
|---|---|---|---|---|---|---|---|---|
| ***1*** | 0 | 1 | 2 | 3 | 4 | 3 | 2 | 1 |
| ***2*** | 1 | 0 | 1 | 2 | 3 | 4 | 3 | 2 |
| ***3*** | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 3 |
| ***4*** | 3 | 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| ***5*** | 4 | 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| ***6*** | 3 | 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| ***7*** | 2 | 3 | 4 | 3 | 2 | 1 | 0 | 1 |
| ***8*** | 1 | 2 | 3 | 4 | 3 | 2 | 1 | 0 |

Fig. 4. Distance among social strategies.

As an example, we can compute the authorities of eight social strategies (i.e., the eight standing orientations) in Fig. 2(b). Let the social positions of the agents be shown as Fig. 3(a), then the authorities of the social strategies are shown as Fig. 3(b). We can find that although the agents that adopt social strategy 4 are agent $b$ and agent $d$, whose social positions are both junior to agent $e$, but the authority of social strategy 4 is more than the social strategy 2 that covers agent $e$.

*Definition 8:* The difference between two social strategies $i$ and $j$ in our case can be defined as the angular distance of their represented standing orientations

$$DL_{ij} = \begin{cases} |j - i|, & \text{if } |j - i| \le 4 \\ 8 - |j - i|, & \text{if } |j - i| > 4. \end{cases} \tag{18}$$

Therefore, the difference among the social strategies in our case is shown as Fig. 4.

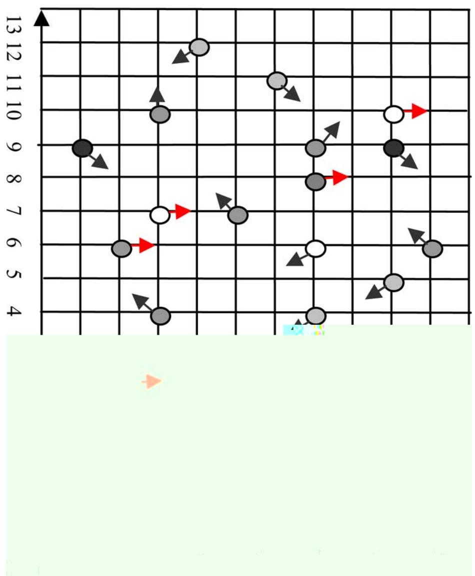


Fig. 5. Agent social strategies with $P_S = 135$. Rank(1) $= 3$, Rank(2) $= 5$, Rank(3) $= 13$, Rank(4) $= 12$, Rank(5) $= 0$, Rank(6) $= 7$, Rank(7) $= 12$, and Rank(8) $= 9$. $P_S = 178/7$.

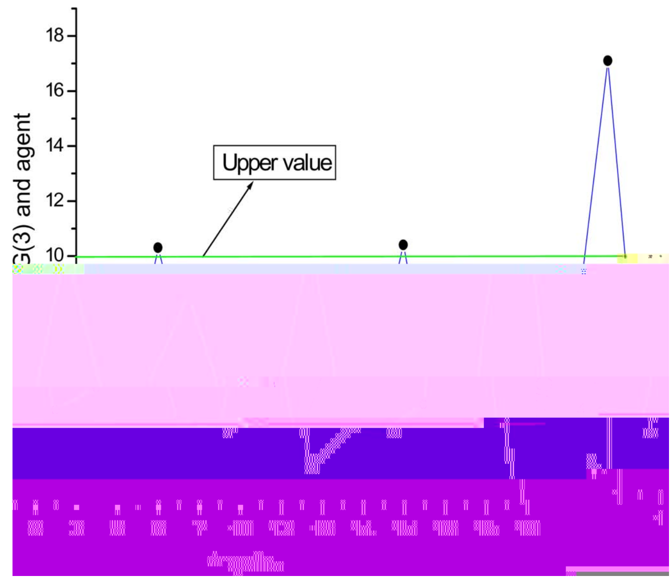


Fig. 6. $\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}$ distribution of other agents to the overlay group of social strategy 3.

### B. Case Study for Sequential Collective Diffusion

*1) Case Demonstrations:* Let the social strategies of $G$ and $a$ before diffusion be $m$ and $n$, respectively, $1 \le m, n \le 8$. If $m \ge n$, the change of the standing orientation of $a$ will be clockwise; if $m < n$, the change of the standing orientation of $a$ will be anticlockwise. Therefore, on the basis of (6), we can design the diffusion criterion of the case in (19), shown at the bottom of the page, where $s'_a$ is the new social strategy of $a$ after one step in the simulation.

We examined several cases to demonstrate and test our model. Fig. 5 is one such case, where social strategy 3 has the strongest authority. So we compute $\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}$ between other agents and the overlay group of social strategy 3. The distribution of $\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}$ is shown as Fig. 6. As a trial, we set $\xi$ and $\eta$ to 10 and 4, respectively. This yields diffusion from $G(3)$ to other agents according to (19) and Algorithm 1, and the progress of diffusion is shown in Figs. 7–9. $P_S$ does not change from Fig. 9,

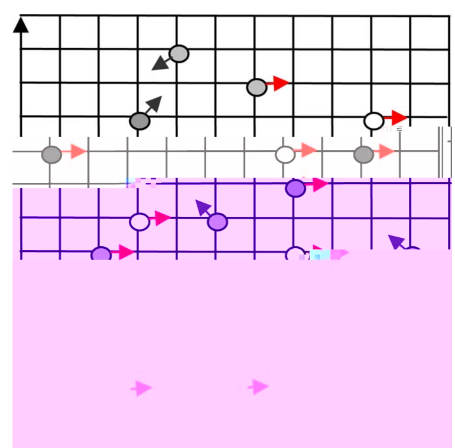

Fig. 7. Agent social strategies after the first-round diffusion from $G(3)$ to other agents, where Rank(1) $= 0$, Rank(2) $= 3$, Rank(3) $= 31$, Rank(4) $= 2$, Rank(5) $= 0$, Rank(6) $= 4$, Rank(7) $= 12$, and Rank(8) $= 9$. $P_S = 393/5$.

$$s'_a = \begin{cases} m, & \text{if } \xi \le \mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G} \\ \left.\begin{array}{ll} \left\lceil 2m - 8 - n + \dfrac{\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}}{\xi} \times (n - m + 8)\right\rceil, & \text{if } m - n > 4 \\ \left\lceil n + \dfrac{\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}}{\xi} \times (m - n)\right\rceil, & \text{if } 0 \le m - n \le 4 \\ \left\lfloor n - \dfrac{\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}}{\xi} \times (n - m)\right\rfloor, & \text{if } -4 \le m - n \le 0 \\ \left\lfloor 2m + 8 - n - \dfrac{\mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G}}{\xi} \times (m - n + 8)\right\rfloor, & \text{if } m - n < -4 \end{array}\right\}, & \text{if } \eta \le \mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G} \le \xi \\ n, & \text{if } \mathrm{IF}_{G\to a}/\mathrm{CF}_{a\to G} < \eta \end{cases} \tag{19}$$

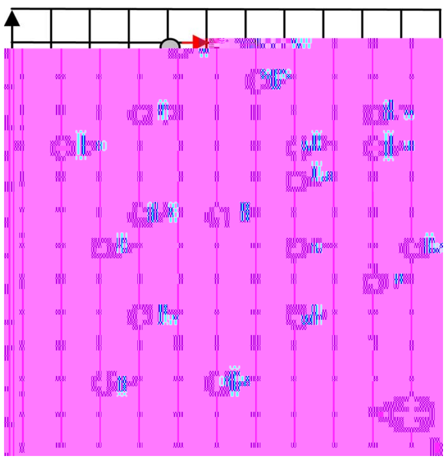

Fig. 8. Agent social strategies after the second-round diffusion from $G(3)$ to other agents, where $\text{Rank}(1) = 0$, $\text{Rank}(2) = 0$, $\text{Rank}(3) = 49$, $\text{Rank}(4) = 0$, $\text{Rank}(5) = 0$, $\text{Rank}(6) = 0$, $\text{Rank}(7) = 12$, and $\text{Rank}(8) = 0$. $P_S = 447$.

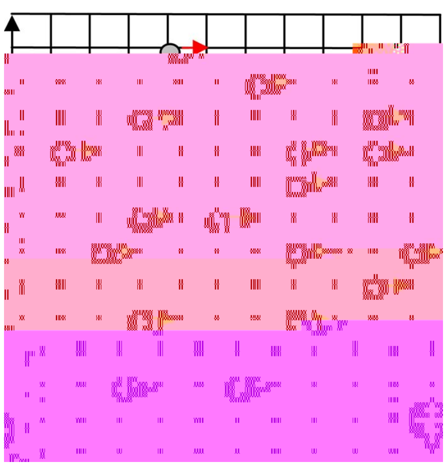

Fig. 9. Agent social strategies after the third-round diffusion from $G(3)$ to other agents, where $\text{Rank}(1) = 0$, $\text{Rank}(2) = 0$, $\text{Rank}(3) = 49$, $\text{Rank}(4) = 0$, $\text{Rank}(5) = 12$, $\text{Rank}(6) = 0$, $\text{Rank}(7) = 0$, and $\text{Rank}(8) = 0$. $P_S = 447$.

so diffusion process is finished. The total number of diffusion steps is 3. From Fig. 9, we can see that the standing orientations of all agents are identical after three diffusion steps, except for the agent in the bottom right corner.

Next, we change the values of $\xi$ and $\eta$. At first, we decrease them step by step for four cases, and then increase them step by step for another four cases. The diffusion results are given in Table I. From Table I, we can see that more steps are needed to reach diffusion termination when $\xi$ and $\eta$ increase. Therefore, we should set $\xi$ and $\eta$ to match the actual situation being simulated.

*2) Analyses for the Case Demonstrations:* Our proposed model shows us that the social strategy of a superior agent may be changed by the social strategy shared by many agents, which is an example of collective insurgent diffusion. The case demonstration showed that it was possible for one agent of quite high position to retain its social strategy even after all other agents have adopted the same social strategy, i.e., the counteracting force of the outlier agent overrides the collective diffusion force of all other agents. Moreover, if we increase the social position of the outlier agent significantly, it is possible that its social strategy will diffuse to all other agents, which is an example of collective elite diffusion.

TABLE I
DIFFUSION RESULTS FOR DIFFERENT $\xi$ AND $\eta$

| CASE | $\xi$ | $\eta$ | DIFFUSION ROUNDS | $S$ | $P_S$ |
|---|---|---|---|---|---|
| 2 | 2 | 0.5 | 2 | 3 | 1159 |
| 3 | 4 | 1 | 2 | 3 | 1159 |
| 4 | 6 | 2 | 3 | 3 | 1159 |
| 5 | 8 | 3 | 3 | 3,5 | 447 |
| **1** | **10** | **4** | **3** | **3,5** | **447** |
| 6 | 12 | 6 | 4 | 3,8 | 447 |
| 7 | 14 | 8 | 4 | 3,8 | 447 |
| 8 | 16 | 10 | 5 | 3,8 | 447 |
| 9 | 18 | 12 | 7 | 3,8 | 447 |

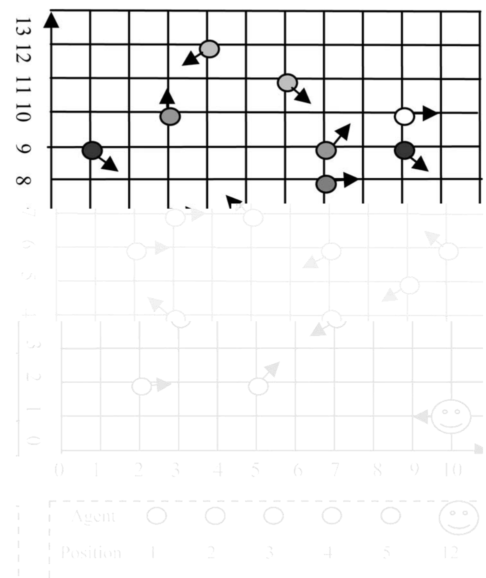

Fig. 10. Initial case, where $P'_S = 12.9$ and $\text{CASS} = 5$.

We note that real-world situations are often characterized by multiple strategies. For example, a commanding officer may walk among his troops in formation, and so has a completely different orientation to everyone else. The troops understand the situation and so unify their own orientations and do not blindly track the officer's orientation. In another simple example, the commander has an absolute power and can order all soldiers to follow his direction; the soldiers have to obey the order.

### C. Case Study for Concurrent Form

Fig. 10 is a case for testing the concurrent mechanism, where there are 19 agents with different social strategies (standing orientations). From the top left to the bottom right, we can number the agent as $a_1, a_2, \ldots, a_{19}$. Now we compute the change probabilities of all agents, respectively, according to (9), (11), and (14), which are shown in Table II. In Table II, the value denotes the social strategy with the highest probability. Originally, the inclination process in Algorithm 2 should be progressed according to (19). Now, for simplification reason,

TABLE II
INCLINED SOCIAL STRATEGIES WITH THE HIGHEST CHANGE PROBABILITIES IN THE INITIAL CASE

| Agent | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $prob_i^s$ | $s_4$ | $s_3$ | $s_4$ | $s_4$ | $s_4$ | $s_3$ | $s_4$ | $s_3$ | $s_3$ | $s_3$ |
| $prob_i^s*$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_3$ | $s_4$ | $s_4$ | $s_3$ | $s_3$ |
| $prob_i^s**$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ |
| Agent | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ | $a_{16}$ | $a_{17}$ | $a_{18}$ | $a_{19}$ | |
| $prob_i^s$ | $s_3$ | $s_3$ | $s_6$ | $s_6$ | $s_3$ | $s_6$ | $s_3$ | $s_3$ | $s_3$ | |
| $prob_i^s*$ | $s_4$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_4$ | $s_3$ | $s_4$ | |
| $prob_i^s**$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | |

TABLE III
INCLINED SOCIAL STRATEGIES WITH THE HIGHEST $\mathrm{prob}_i^s*$ IN FIG. 11

| Agent | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $prob_i^s*$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ |
| Agent | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ | $a_{16}$ | $a_{17}$ | $a_{18}$ | $a_{19}$ | |
| $prob_i^s*$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | $s_4$ | |

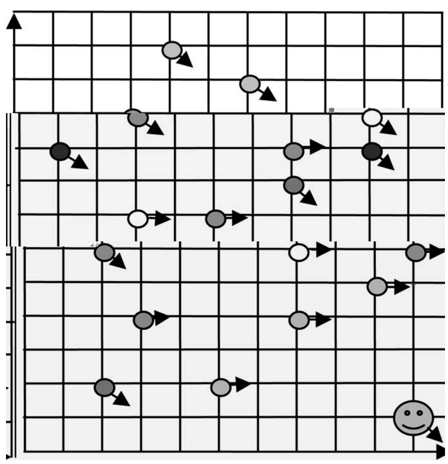

Fig. 11. Agent system after the first diffusion by choosing $\mathrm{prob}_i^s*$. CASS $= 4$ and $P_s' = 622.8$.

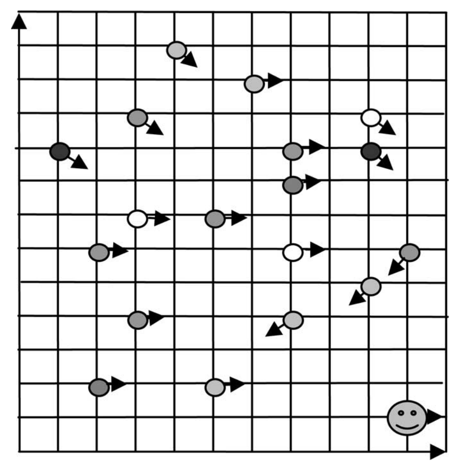

Fig. 12. Agent system after the first diffusion by choosing $\mathrm{prob}_i^s$. CASS $= 4$ and $P_s' = 197.5$.

we set $\xi = \eta = 0$ in (19). Thus, the new strategy of agent $i$ will be fully changed as $s^{\otimes}$.

1) If we select $\mathrm{prob}_i^s **$, the diffusion can be finished within one step, and the diffusion result is that all agents adopt $s_4$.
2) If we select $\mathrm{prob}_i^s*$, the diffusion result after the first step is shown as Fig. 11. Now we make the second step diffusion for Fig. 11 according to $\mathrm{prob}_i^s*$, the results are shown in Table III. From Table III, we can see that now the diffusion is finished.
3) If we select $\mathrm{prob}_i^s$, the diffusion result after the first step is shown as Fig. 12. Then, we make the second step diffusion for Fig. 12 according to $\mathrm{prob}_i^s$. The results are shown in Table IV. Therefore, now the diffusion is finished.

Analyses for the test results:

1) From the earlier results, we can see that it only needs one step for the diffusion if we select $\mathrm{prob}_i^s **$. When we select $\mathrm{prob}_i^s **$, we consider the concurrent mechanism and the unification trend. Thus, the social strategies of all agents can reach unification shortly.
2) When we select $\mathrm{prob}_i^s*$ to decide the change probability of social strategy, we need two steps to reach the status of unification. After the first step, the number of social strategies in the system is 2.
3) When we select $\mathrm{prob}_i^s$ to decide the change probability of social strategy, we also need two steps to reach the status of unification. The number of steps is equal to those of change probability with $\mathrm{prob}_i^s*$. However, when the first step is finished, the performance of the system with $\mathrm{prob}_i^s*$ is more than the one with $\mathrm{prob}_i^s$. Therefore, $\mathrm{prob}_i^s*$ considers the concurrent mechanism and unification trend better than the $\mathrm{prob}_i^s$.

*Summary:* We can also demonstrate our model in some other cases, which is shown in Table V. Therefore, for the performance criterion, $\mathrm{prob}_i^s ** > \mathrm{prob}_i^s* > \mathrm{prob}_i^s$, where "$>$" denotes "*performs better than*."

## V. RELATED WORK

The research of this paper is related to the collective motion, simulation, and social behavior of multiagents, such as social

TABLE IV
INCLINED SOCIAL STRATEGIES WITH THE HIGHEST $\mathrm{prob}_i^s$ IN FIG. 12

| Agent | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $prob_i^s$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ |
| Agent | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ | $a_{16}$ | $a_{17}$ | $a_{18}$ | $a_{19}$ | |
| $prob_i^s$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | |

TABLE V
TEST RESULTS FOR OTHER CASES, WHERE $P'_S(i)$ DENOTES $P'_S$ AFTER THE $i$TH STEP DIFFUSION

| Case | Change Probability | *Steps* | $P_S(1)$ | $P_S(2)$ | $P_S(3)$ | $P_S(4)$ | $P_S(5)$ | $P_S(6)$ |
|---|---|---|---|---|---|---|---|---|
| 1 | $prob_i^s$ | 2 | 85 | 231 | / | / | / | / |
| | $prob_i^s*$ | 2 | 114 | 231 | / | / | / | / |
| | $prob_i^s**$ | 1 | 231 | / | / | / | / | / |
| 2 | $prob_i^s$ | 5 | 34 | 139 | 291 | 448 | 981 | / |
| | $prob_i^s*$ | 4 | 112 | 298 | 361 | 981 | / | / |
| | $prob_i^s**$ | 2 | 242 | 981 | / | / | / | / |
| 3 | $prob_i^s$ | 6 | 12 | 132 | 189 | 208 | 302 | 994 |
| | $prob_i^s*$ | 4 | 34 | 189 | 245 | 994 | / | / |
| | $prob_i^s**$ | 2 | 213 | 994 | / | / | / | / |
| 4 | $prob_i^s$ | 3 | 24 | 109 | 341 | / | / | / |
| | $prob_i^s*$ | 2 | 98 | 341 | / | / | / | / |
| | $prob_i^s**$ | 1 | 341 | / | / | / | / | / |
| 5 | $prob_i^s$ | 6 | 12 | 45 | 176 | 221 | 304 | 981 |
| | $prob_i^s*$ | 4 | 23 | 75 | 331 | 981 | / | / |
| | $prob_i^s**$ | 3 | 103 | 231 | 981 | / | / | / |

cognitive modeling of social and collective action. Generally speaking, the related work can be categorized as follows.

1) *The collective motion of multiagents*: The collective motion of multiagents has been receiving much attention in many research fields [1]–[7]. The previous benchmark works on collective motion of multiagents focused on the strategy control of individual agents; an individual agent may sense and adjust its own strategy to keep pace with the neighboring agents. In the collective motion of multiagents, each agent can select any arbitrary initial strategies to behave; with the passage of time, each agent acts solely on the basis of its own local perception of the world and imitates the average strategy of its neighbors; thus, the collective synchronization may be received.
2) *On the multiagent simulation, the one for crowd of virtual human and social behaviors is attracting much attention* [24], [25]. The related work mainly focused on presenting frameworks or virtual scenarios for simulating the human and social behaviors, and the research results are always related on some specific applications, such as evacuation systems, training systems. The goal of related work is to reproduce realistic (or near realistic) scenarios or environments of social human activities. Therefore, the architecture and framework are always the key issues.
3) *Diffusion is always explored in the social science* [14]–[16]: In these works, the authors mainly investigate the large scale of penetration of some social phenomena, such as knowledge diffusion, innovation diffusion. Therefore, the work of diffusion in social science care about the social phenomena, but not the modeling and simulation based on multiagents.
4) *On the social aspects of multiagent systems, modeling and analyses on the organizations and norms are studied in the research community* [23], [26]–[31]. These related work focus on the virtual organizations of multiagent societies based on norms or institutions; especially, the formal semantics of organization should be defined to implement a social multiagent system.

In summary, these works do not make systemic research on the collective diffusion mechanism in large scale of multiagent systems; moreover, the concurrent form of diffusion has not been modeled in the multiagent community.

## VI. DISCUSSION AND CONCLUSION

In this paper, we present a model for the collective social strategy diffusion in multiagent societies. The two kinds of diffusions, collective insurgence and elite diffusions, are modeled by setting the variables differently in our formal framework and demonstrated by the case studies. Moreover, this paper also explores the concurrent form of collective diffusion with unification trend. In the concurrent diffusion, many diffusion processes will take place simultaneously, and an agent's social strategy is determined by not only the diffusion that bears on itself but also other diffusion processes that bear on other agents.

This paper is mainly focused on the spatial diffusion model, which assumes that all agents locate in a euclidian space, and the social distance between agents can be measured as a geographical one. Regarding the future work, we are currently working on the development and application of the model in real society. We will improve our model's adaptation for the real society diffusion by involving more social and cognitive factors.

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