

Environment-Driven Social Force Model: Lévy Walk Pattern in Collective Behavior

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Abstract

Animals in social foraging not only present the ordered and aggregated group movement but also the individual movement patterns of Lévy walks that are characterized as the power-law frequency distribution of flight lengths. The environment and the conspecific effects between group members are two fundamental inducements to the collective behavior. However, most previous models emphasize one of the two inducements probably because of the great difficulty to solve the behavior conflict caused by two inducements. Here, we propose an environment-driven social force model to simulate overall foraging process of an agent group. The social force concept is adopted to quantify the conspecific effects and the interactions between individuals and the environment. The cohesion-first rule is implemented to solve the conflict, which means that individuals preferentially guarantee the collective cohesion under the environmental effect. The obtained results efficiently comply with the empirical reports that mean the Lévy walk pattern of individual movement paths and the high consistency and cohesion of the entity group. By extensive simulations, we also validate the impact of two inducements for individual behaviors in comparison with several classic models.

1 Introduction

The environment and the conspecific effects simultaneously induce the complex behaviors of an animal group [Pillot *et al.*, 2011]. Facing to the heterogeneous and uncertain environment in social foraging, animals adjust the movement speeds and the tortuosity of the movement paths [Benhamou, 2007] to maximize the foraging efficiency. This tortuous paths generally show the Lévy walk pattern which is characterized as the power-law frequency distribution $P(l) \sim l^{-\alpha}$ of flight lengths (l) with $1 < \alpha \leq 3$ [Turchin, 1998; Viswanathan *et al.*, 1996]. Meanwhile, the animal group can still keep the collective cohesion and alternately present the order state or the disorder state with moving out of or into the food patches [Viswanathan *et al.*, 1996]. The modeling method to reproduce the collective behavior has been widely investigated

[Couzin and Krause, 2003; Han *et al.*, 2011; Li and Jiang, 2014]. However, in most studies, each agent behaves depending on one of the two inducements and cannot simultaneously perform the group-level and individual-level behaviors. For instance, some models focus on how the conspecific effects lead to the consistent behavior of an agent group and individual moves according to the average moving directions and speeds of neighbors [Reynolds, 1987; Vicsek, 1995]. Consequently, agents move in the less tortuous line. In some other studies, the environment effects determine the destination or moving direction of each agent while the group consistency in collective movement are generally be ignored [Han *et al.*, 2011; Viswanathan *et al.*, 1999].

However, there is a conflict between the effects of food and conspecifics [Dumont and Boissy, 2000; Sibbald and Hooper, 2004]. On one hand, the heterogeneous food distribution in the system drives individuals to access more and better food resources. On the other hand, the interactions between conspecifics constrain individual behaviors to stay in the group [Ballerini *et al.*, 2008; Reynolds, 1987]. Therefore, the solution to the conflict may be the core to form the mechanism of individual Lévy walk pattern in collective behavior. According some real field experiments on sheep that a single individual never leave the group to pursue better food [Dumont and Boissy, 2000; Sibbald and Hooper, 2004], a cohesion-first rule is suggested: individuals will guarantee the collective cohesion preferentially.

Thus, this paper mainly aims at simulating the individual-level Lévy walk pattern and the group-level consistency and cohesion in an agent-based model. We propose an environment-driven social force model to describe individual behavior strategies. The social force concept is used to quantify the physics and psychology motivations to guide individual behaviors [Helbing and Molnar, 1995]. The friction, pull force, and repulsion-attraction force are generally applied to characterize individual interactions with the environmental food and the conspecifics [Li and Jiang, 2014]. Individuals make own behavior decision by estimating the values and the directions of the three forces autonomously and evaluating which effect is firstly considered based on the cohesion-first rule. Individuals are the forager on a two dimensional square and the food resources are patchily distributed. For the patchy environment, our model involves a switching strategy between the searching paradigm and the

feeding paradigm to address environmental mutation, which means the transition from food area to waste land or vice versa [Fauchald and Tveraa, 2006; Nabe-Nielsen, 2013].

By extensive simulations, our model can achieve two objectives: 1) individual movement pattern present the characteristics of Lévy walks and 2) the group still keeps collective characteristics under the influence of environment. Meanwhile, by the test on the population density, we find that the patchily distributed food causes the conspecific competition resulting in disorder and dispersion while the conspecific cooperation can emerge with the order movement of all the agents if the group moves between patches.

2 Model Design

2.1 Individual and Foraging Environment

The individuals in our model have some general capabilities like the seeing, perceiving, moving and feeding. Individuals are regarded as the agents that are autonomous and individual entities. In Figure 1(a), an agent is shown as an arrow that points to moving direction or facing direction and the origin represents the location of agent. Agent a is characterized by the current location \mathbf{x}_a and velocity \mathbf{v}_a . The agent has a sight distance r_s and a two-dimensional view range $[-\pi, \pi]$ [Moussaïd *et al.*, 2011] which are represented by the shadow area in Figure 1(a). The food resources can be only perceived by eyes. However, agents can locate the companions within the perception domain, which are the circular regions with the radius r_c . The perception domain is divided into the repulsion zone, the comfortable zone and the attraction zone [Couzin *et al.*, 2002]. The repulsion zone is a circle area with radius d_c . The comfortable zone and attraction zone are both annular areas with the width $d_s - d_c$ and $r_c - d_s$, respectively.

The environment is represented by two-dimensional $N \times N$ grids with nonperiodic boundary conditions. The patchiness is proposed to describe the heterogeneous distribution of resources in the real world [Fauchald and Tveraa, 2006]. The pixels (grids) with food are nested in patches. The patches denoted as circles of radius r_p are distributed discretely and randomly. The pixels are indivisible food units that fill patches continuously. An arbitrary pixel can be located by a unique coordinate \mathbf{p}_i . An integer value A_i denotes the amount of food at the Pixel i (Figure 1(b)). The maximum amount A_m of food in a pixel is constant. As the agents consume the food, A_i will decrease to zero. Here, the food cannot be replenished.

2.2 Force Representation

In our model, we consider that the agents hope to keep a comfortable distance with companions and are also attracted by the preferred food. These motivations are regarded as the interactions of agents with the companions and the food, which are distinguished into three types of forces based on social force model. The interactions among these three forces determine the moving directions and speeds of agents.

Friction

Li and Jiang in [2014] consider that the food at the location where the agent stands presently will obstruct it to leave. This

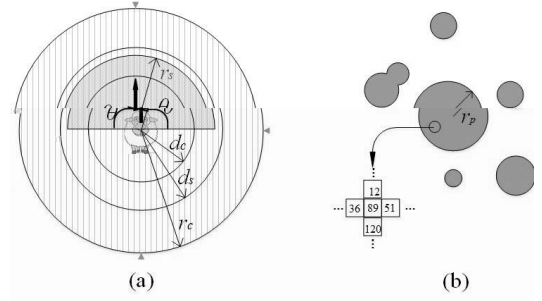


Figure 1: (a) and (b) characterize the individual and foraging environment, respectively.

effect is similar to “Friction” in Newtonian Mechanics. The friction force emerges only if the agent tends to move away from current location with food. The value of friction is linearly correlated with the amount of food at current pixel. The direction of friction is opposite to the direction of velocity. Thus, the friction is given by

$$\mathbf{F}_f(a, i) = \mu \times A_i \times \frac{-\mathbf{v}_a'}{|\mathbf{v}_a'|}. \quad (1)$$

Here, $\mathbf{F}_f(a, i)$ is the vector of friction on Agent a at Pixel i . The factor μ is the “Friction Coefficient” considered as the magnitude of friction and is identical for each agent.

Pull Force

In social foraging, the foundational objective of agents is to access more food. Thus, the agent tends to move to the pixels with abundant food instinctively. The pull force quantifies this tendency and drives the agent to the objective pixel. The location of objective pixel and the amount of food determine the direction and the value of pull force. Pull force is computed by a function with natural exponent [Turchin, 1998].

$$\mathbf{F}_p(a, i) = \mu_p \times \frac{e^{\frac{A_i}{A_m}} - 1}{e - 1} \times \frac{\mathbf{p}_i - \mathbf{x}_a}{|\mathbf{p}_i - \mathbf{x}_a|}, i \in \mathcal{A}. \quad (2)$$

The factors \mathbf{p}_i and \mathbf{x}_a are the location coordinates of Pixel i and Agent a . The factor μ_p determine the magnitude of $\mathbf{F}_p(a, i)$.

Repulsion-attraction Force

Each agent in the group should follow two rules: (1) avoiding collision with the near neighbors, (2) approaching the group center to avoid being alone [Couzin *et al.*, 2002; Reynolds, 1987]. The repulsion-attraction force is applied to explain the conspecific effects mathematically. Repulsion-attraction force means that the repulsion force and the attraction force are distinguished by the distance between the focal agent and other agents [Jiang, 2009; Li and Jiang, 2014]. The attraction force makes agents approach each other. The repulsion force makes agents away from each other.

$$\begin{aligned} \mathbf{F}_c(a, b) &= \mu_c \times \frac{e^{\frac{d_s - d}{d_c}} - 1}{e - 1} \times \frac{\mathbf{x}_b - \mathbf{x}_a}{|\mathbf{x}_b - \mathbf{x}_a|}, d < d_c; \\ \mathbf{F}_c(a, b) &= \mu_c \times \frac{e^{\frac{d_s - d}{d_s}} - 1}{e - 1} \times \frac{\mathbf{x}_b - \mathbf{x}_a}{|\mathbf{x}_b - \mathbf{x}_a|}, d_s < d < r_c; \\ \mathbf{F}_c(a, b) &= 0, d_c < d < d_s. \end{aligned} \quad (3)$$

Here, $F_e(a,b)$ indicates the repulsion force or the attractive force between Agent a and Agent b that is one of neighbors in the perception domain of Agent a . Moreover, α is used to adjust the magnitude of $F_e(a,b)$.

The repulsion-attraction force of Agent a at p_i , $F_e(a|i)$, is the vector sum of $F_e(a,b)$. Meanwhile, $F_e(a|i)$ also presents either the repulsion force or the attraction force. $F_e(a|i)$ is determined by perceiving the number of neighbors in the three zones. If the sum of neighbors in the repulsion zone and the comfortable zone is greater than k , $F_e(a|i)$ can be computed by the vector sum of repulsion forces from agents in the repulsion zone of Agent a . Otherwise, $F_e(a|i)$ will be equal to the vector sum of attraction forces from agents in the attraction zone. Here, k is the threshold to judge that Agent a locates at close to group center or the border of the group.

$$F_e(a|i) = \sum_{b \in O_a} F_e(a,b) \quad (4)$$

O_a is the set of neighbors of Agent a deriving from either the repulsion zone or the attraction zone. The smaller the value of $F_e(a|i)$ is, the more comfortable the Pixel i is for Agent a .

2.3 The Model

Agents perform composite movement pattern to enhance search efficiency that includes extensive search between patches and intensive search within patches [Benhamou, 2007; Nonaka and Holme, 2007]. The extensive search means that individuals travel fast and straight with large displacements between patches, but the intensive search indicates the low velocities and large turn angles moving within patches. Therefore, individuals present two different movement paradigms. The continuous foraging process in this paper is considered as a switching strategy between two movement paradigms: searching paradigm and feeding paradigm [Benhamou, 2007; Fauchald and Tveraa, 2006; Nabe-Nielsen, 2013; Nonaka and Holme, 2007].

Two decision processes are determined corresponding to the two motion paradigms. In the searching paradigm, agents make decision through computing resultant force, namely, the force analysis. If each agent reaches the force balance, all agents will have identical velocity value and direction. In the feeding paradigm, agents are hard to reach a force balance because of the heterogeneously distributed food. However, agents can feed only if they are motionless. The method of force analysis is unsuitable for agents' decision-making in the feeding paradigms. Therefore, we consider that agents

move to the target pixels directly within a time step, and the forces here are used to select target pixels by comparing the directions and values. Figure 2 shows the critical conditions to switch between the two paradigms.

In the feeding paradigm, with the consumption of the food at located pixel, the friction of Agent a at the present position cannot resist the repulsion-attraction force. According to the cohesion-first rule, agents try to meet own needs of food under the premise of staying in the group. Figure 3 shows the selecting flow of destination. If agents decide to leave the current pixels, two rules must be followed: (1) the value of friction acting on focal agent is less than the repulsion-attraction force; (2) pixels within sight can provide enough friction for the focal agent to resist the repulsion-attraction force. Each elements of the set O_a that consist of all the pixels within sight of Agent a is tested by comparing the difference value that the value of friction minuses the value of repulsion-attraction force. The Pixel j is selected because of the maximum difference value in O_a . It means that Agent a can enjoy either relatively abundant food or comparatively comfortable position. This selection method avoids that too much preference for food leads to the departure of agents in the group and prevents the fear of social isolation and the decrease of the foraging efficiency. After the selection, agents move to the target pixels directly. Otherwise, Agent a still remains at the current pixel.

In the searching paradigm, the destinations of agents at

In this paper, we assume that these forces are stable in one time interval. Thus, agents can be considered to perform uniformly accelerated motion within one time step. The velocity at next time step is calculated by

$$\mathbf{v}_a^{t+1} = \frac{\mathbf{F}^t(a, i)}{m} \times \Delta t + \mathbf{v}_a^t. \quad (6)$$

Then, the position of Agent

servations of flight distributions of animals [Han *et al.*, 2011].

Then, we test other correlative indexes to explain how the food density influences the behaviors. We conclude the trends of the mean amount of food consumed at one time step, the step length, average velocity, and sinuosity of path with the increase of food density. In Figure 5(a), the food consumed by the agent increases if the food density increases. In Figure 5(b), (c) and (d), the mean step length and average velocity in one time step present an apparent descending trend; on the contrary, the sinuosity of movement path increases. Combining these three graphs, we can see that agents adjust the behavior strategies to accommodate the environment of different food densities. As food density gradually increases, agents present shorter step lengths and larger turn angles, which make the movement path more tortuous, reduce the net displacement and avoid moving out of the current patch. Agents also reduce the velocities to prevent from missing abundant food sites and capture more food as shown in Figure 5a. Therefore, the food distribution is the main factor to drive agents to perform different behaviors according to the environment-driven social force model.

Collective Consistency and Cohesion

In this section, we prove that our model can also present the collective consistency and cohesion of the entire group. The collective consistency is measured by the group polarization [Vicsek, 1995], and the number of neighbors of agents is used to describe the collective cohesion [Davidson and Morris, 2001]. Group polarization means the order degree of alignment and can be obtained by equation (8).

$$\Psi_{group} = \frac{1}{n} \times \left| \sum_{a=1}^n \frac{\mathbf{v}_a^t}{|\mathbf{v}_a^t|} \right|. \quad (8)$$

Here, n is the scale of the group. If the Ψ_{group} is close to 1, the group shows the high order state. Inversely, the group aligns disorderly if Ψ_{group} tends to 0. Here, the high value of group polarization denotes that agents reach an agreement on the moving direction. As shown in Figure 6, the group polarization fluctuates with the amount of food. The change in the lightness of the grey background indicates the different average amounts of food at the current pixels of all agents. Figure 6 shows that the group polarization varies steeply and cannot reach a high value when the background is dark grey (low lightness and more food). However, it is easy to reach a high value of group polarization with relatively smaller fluctuation if the background becomes white (high lightness and less food). Therefore, the group consistency is sensitive to the amount of food. Furthermore, our model confirms the perspective [Buhl *et al.*, 2006] that food shortage facilitates the formation of collective consensus. Due to the uncertainty on food locations, cooperation (order) will provide more competitiveness than the search of the single agent. However, agents begin to seek personal benefit and the competition (disorder) emerges when agents travel into a patch.

The cooperation keeps the agents in an aggregate group, and individuals can split temporally because of the competition [de Jager *et al.*, 2011]. The increased population density will reduce the cost of finding the new patches, but popula-

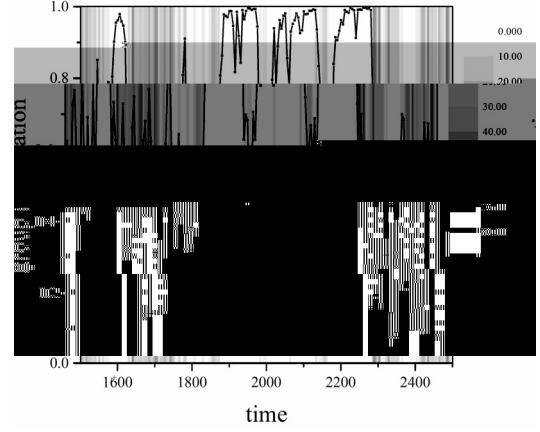


Figure 6: The group polarization of the flock change with the amount of food.

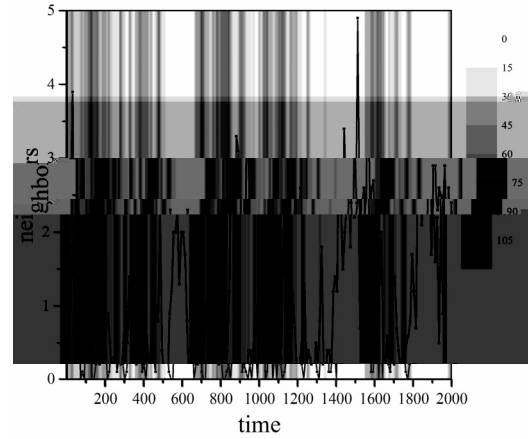


Figure 7: The neighbors around the focal agent change with the amount of food within the constant exploring range of 8 pixels.

tion density will decrease if food is abundant because the communication time between conspecifics in a large group can greatly improve the cost of foraging [Davidson and Morris, 2001]. The food density is the inherent reason. This feature of social foraging can also be presented by our model. As shown in Figure 7, the number of neighbors around an agent changes over time. The severe fluctuation of the broken line means that the value of the number of neighbors hardly becomes stable. It is because agents keep on adjusting the locations for fitness no matter within or between patches. The high value is always along with the low amount of food, and the number of neighbors in the continuous area of less food is higher than the one in the continuous area of abundant food.

Comparison with the Existing Models

In this section, we compare our model with several typical models in the patchy foraging environment. The following is a list of models in simulations.

- Vicsek model (VM) is the self-propelled particles model [Vicsek, 1995]. Agent with unlimited perception domain determines the moving direction depending on average directions of all the members in the system. VM can present high consistency (Table 1).

Table 1: Environment-driven social force model (ESF) compare with SF, VM, LF, and DW on group polarization (GP) and population density (PD, within the constant exploring range of 30 pixels); “/” means that the model does not consider the capability or the condition.

	Sight distance	Capabilities of individuals			
		GP within patch	GP out of patch	PD within patch	PD out of patch

However, because of the absence of the environmental effect, agents hardly change moving direction. Therefore, the flight distribution fails to follow a power-law distribution (Figure 8).

- Social force model without environment (SF) simplifies the collective dynamics based on selective attraction and repulsion proposed by Romanczuk and Schimansky-Geier [2012]. Individuals with limited perception domain determine the moving direction and velocity based on the force analysis. This model can get high value of group polarization and population density (Table 1). However, agents move with the long flights much more than the short flights because agents cannot perform diverse velocities and are generally influenced by balanced forces without the environmental effect (Figure 8).
- Purely statistical Lévy flight model (LF) is applied for the search for randomly distributed target in a large space [Viswanathan *et al.*, 1999]. Individuals determine the moving distance in a random direction according to the probability density function of power-law distribution (Figure 8). Therefore, this model can simulate Lévy Walk patterns of individual (Figure

8) but present low consistency and cohesion (Table 1) without the conspecific effects.

- Deterministic walk model (DW) is proposed by Han *et al.* [2011], in which individuals directly jump to the location of targets (the nearest position with the maximum food). Because of the unlimited sight distance, the flight length depends on the food distribution (Figure 8). In the patchy environment, DW can present the power-law distribution of flight length perfectly (Figure 8). However, this model cannot simulate the consistency and agents are dispersive because of the absence of conspecific effects (Table 1).

Therefore, the environment is the core to induce the diverse flights and tortuous movement paths. The conspecific effects lead to the aggregated and consistent group movement.

5 Conclusion

The main contribution of this paper is to implement the cohesion-first rule in the environment-driven social force model to simulate the collective behavior of a group of agents. The environmental effects drive the agent to perform complex patterns of individual movement and the cohesion-first rule solves the conflict between the conspecific constraint and the food preference. The simulation results accord with the empirical results, which mean the Lévy walks of individuals and the collective characteristics of the entire group.

Furthermore, our model can help to understand the overall movement process of animal groups in social foraging under the influence of the conspecifics and the environment. Moreover, it is hoped that our model can provide a novel perspective in the design of decision-making mechanism.

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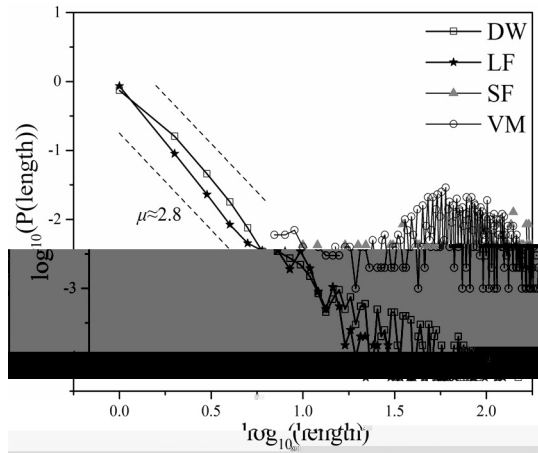


Figure 8: The distribution of flight length from DW, LF, SF, and VM.

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