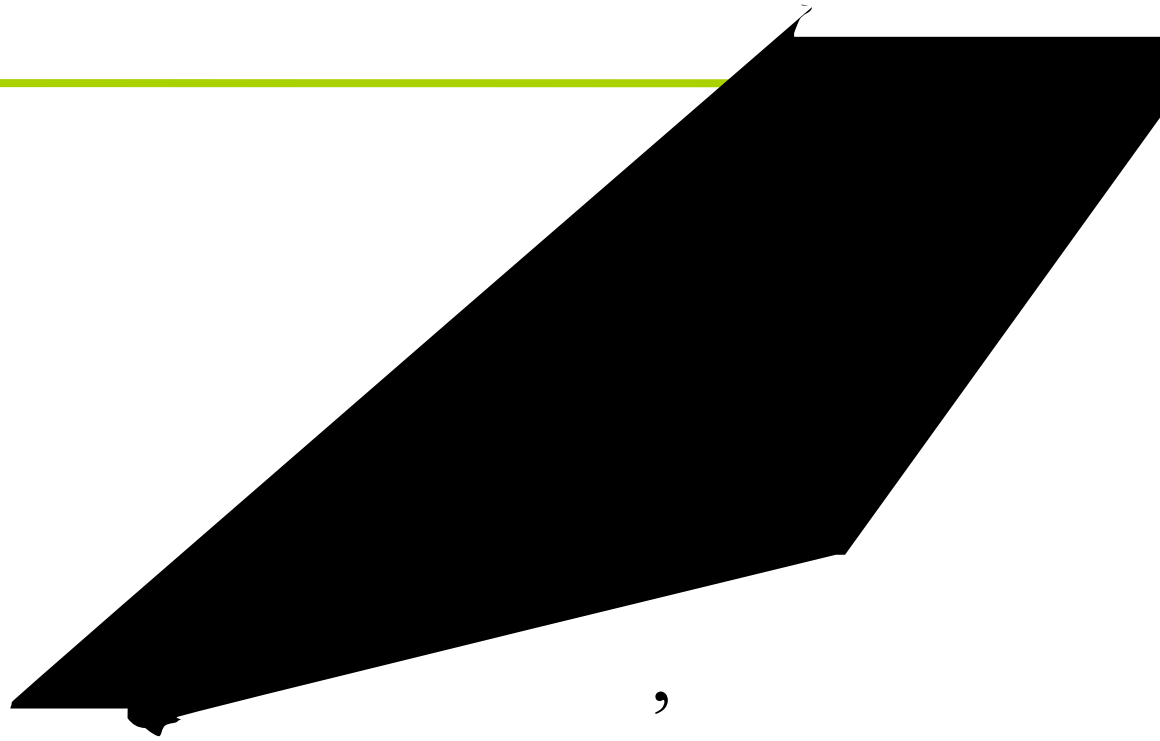
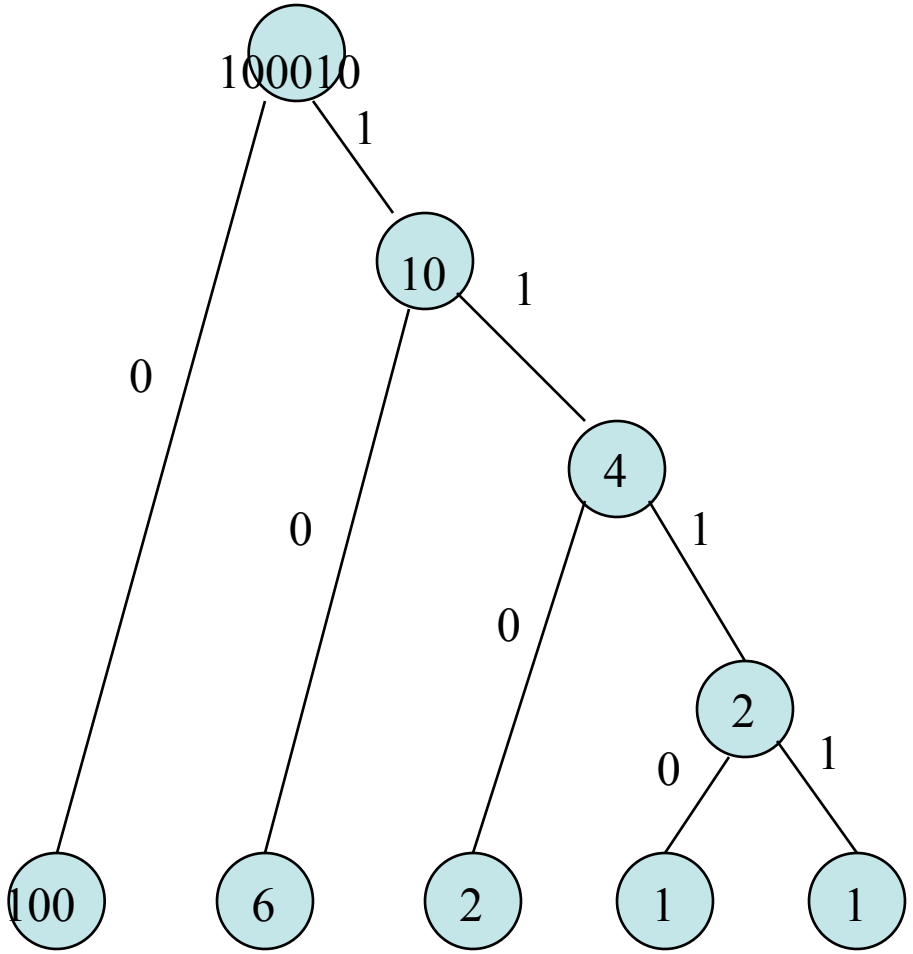
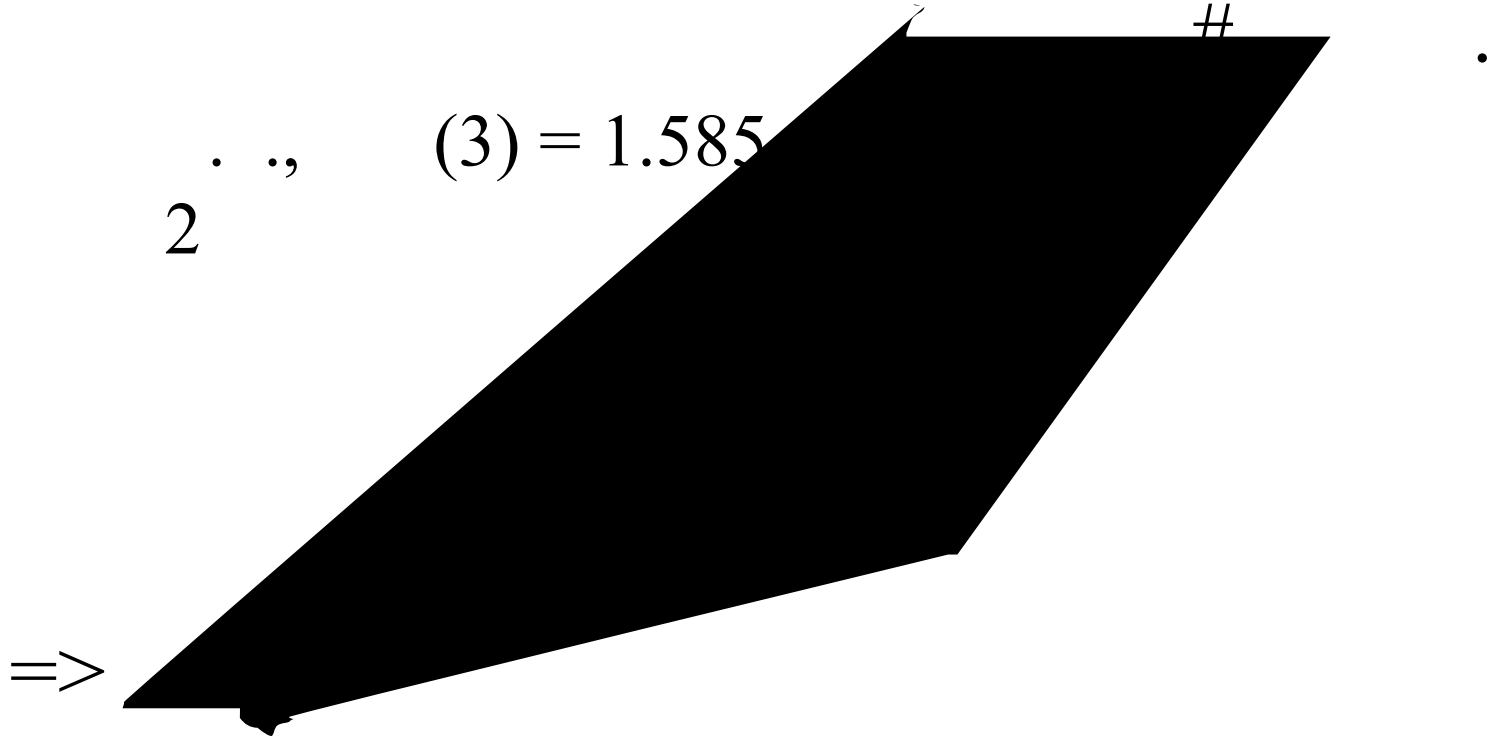
2:  ,

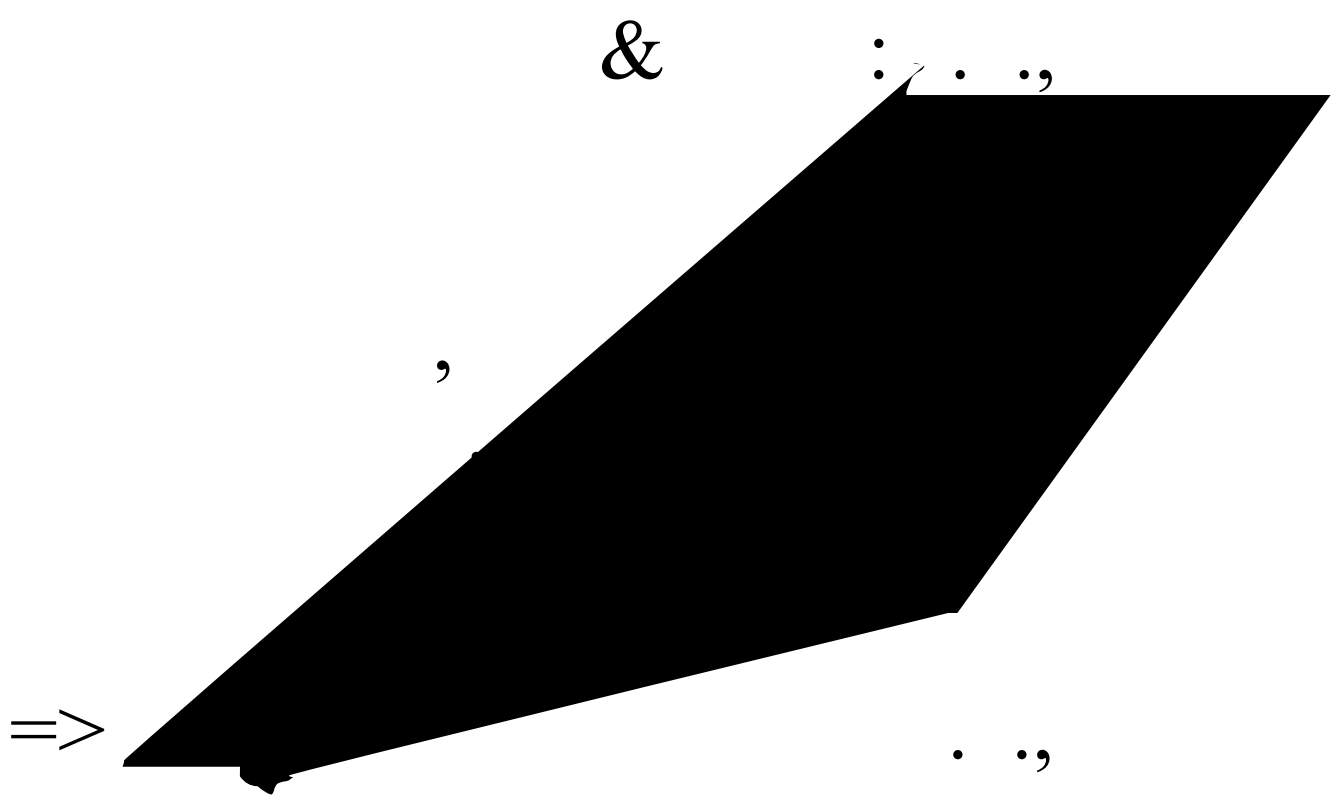
	100000	0
	6	10
	2	110
	1	1110
	1	1111

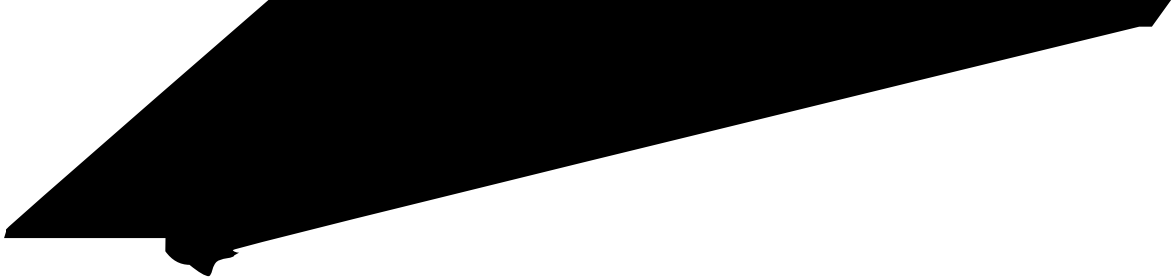


$= 0.9999 \quad 1.0001 + 0.00006 \quad 16668.333$
$+ \quad + 1/100010 \quad 100010$
$\approx \mathbf{0.00}$

$= (100000*1 + \quad)/100010$
$\approx \mathbf{1}$







Encoder

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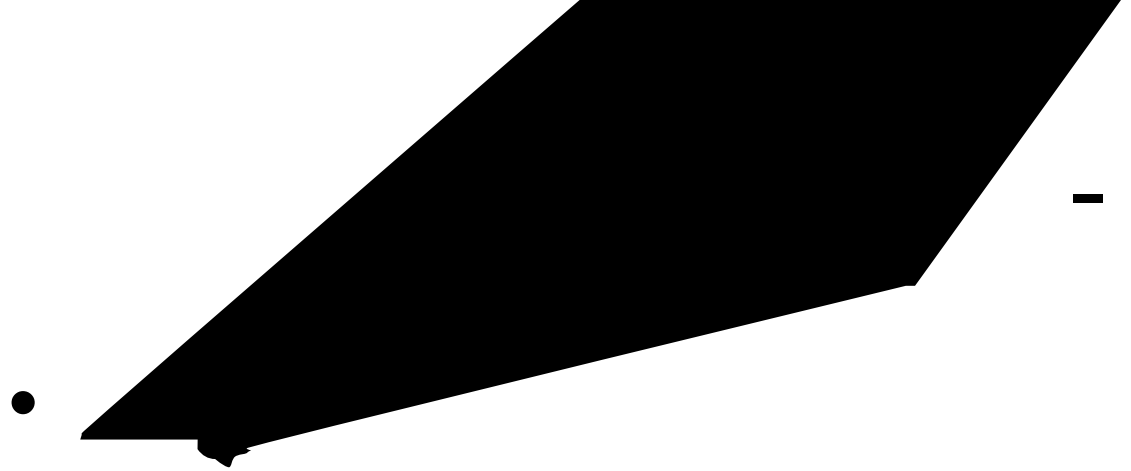
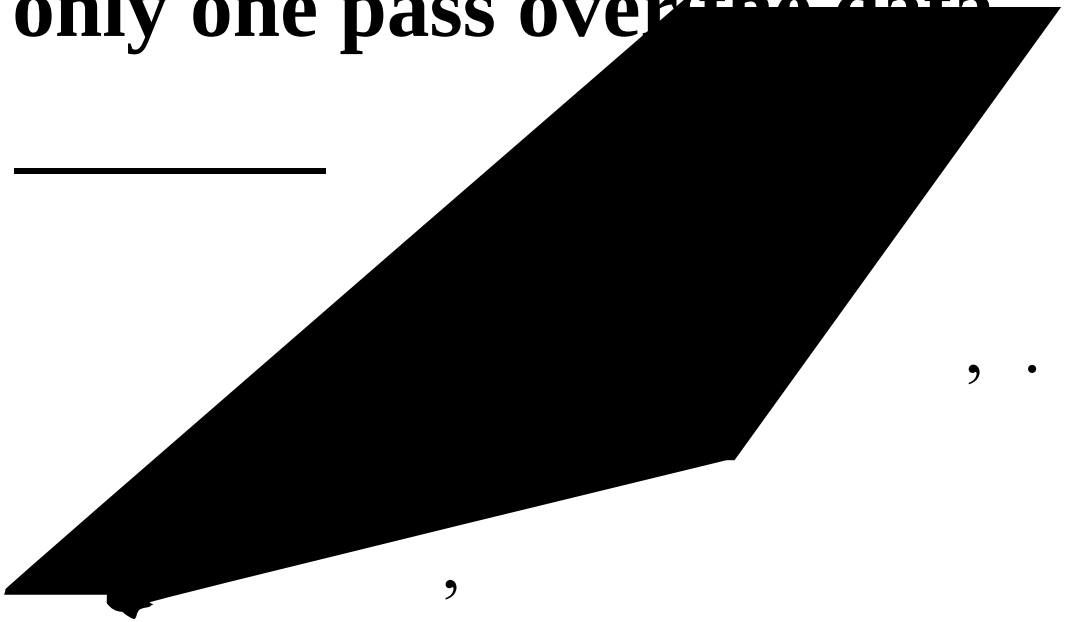


running estimate

– Effective exploitation of locality

& -

– **only one pass over the data**



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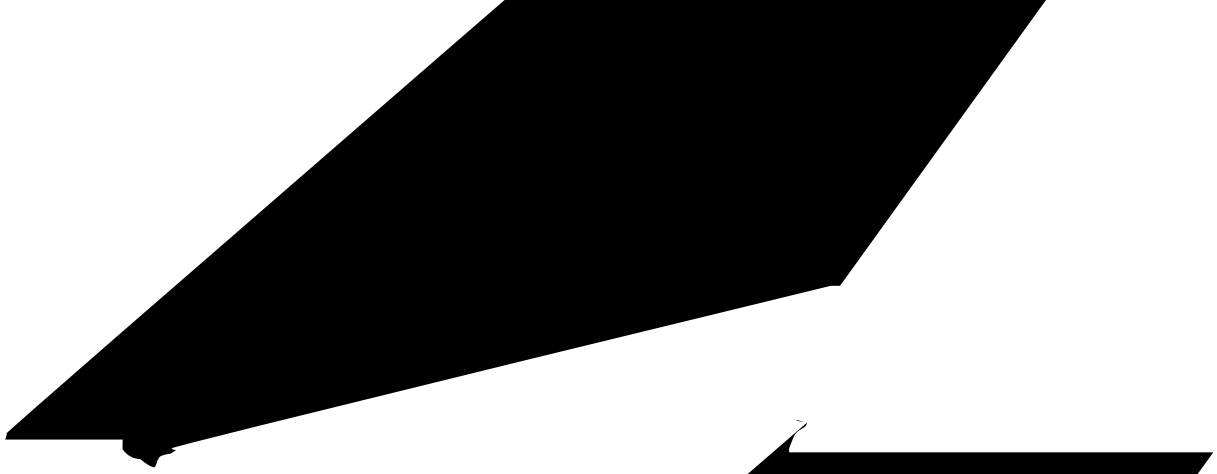
- 

(1978)

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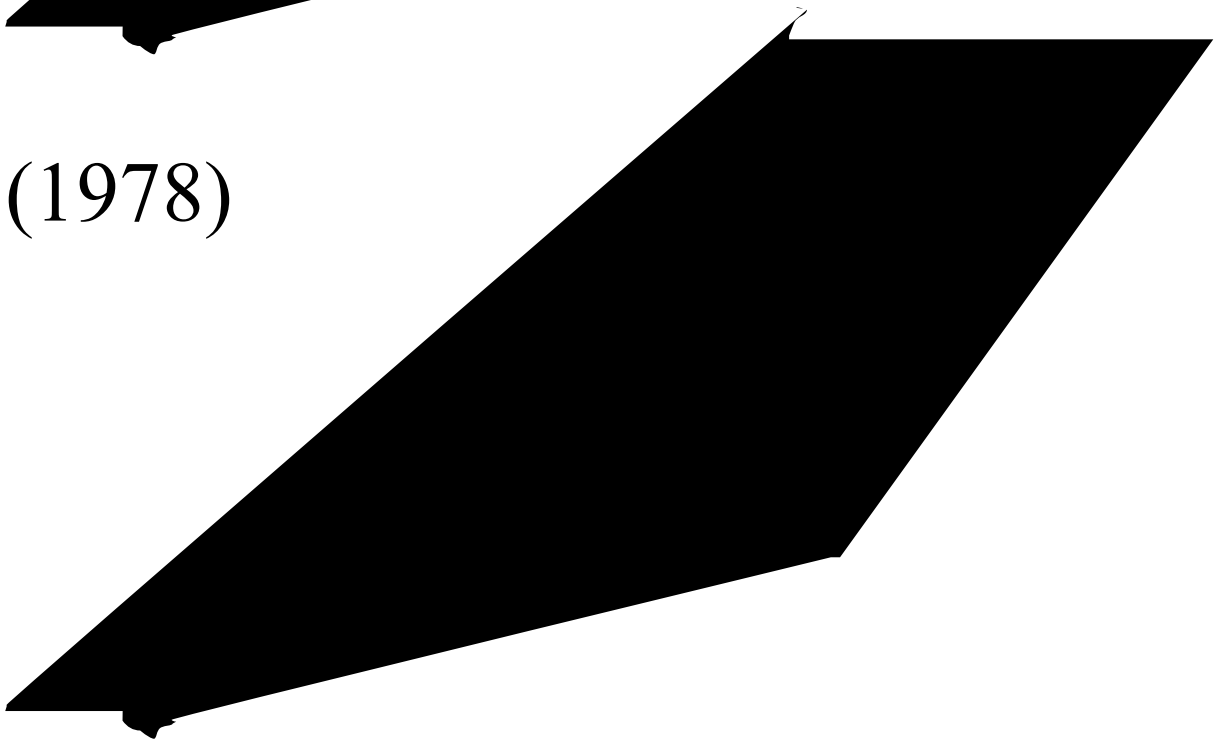
- 

(1987)

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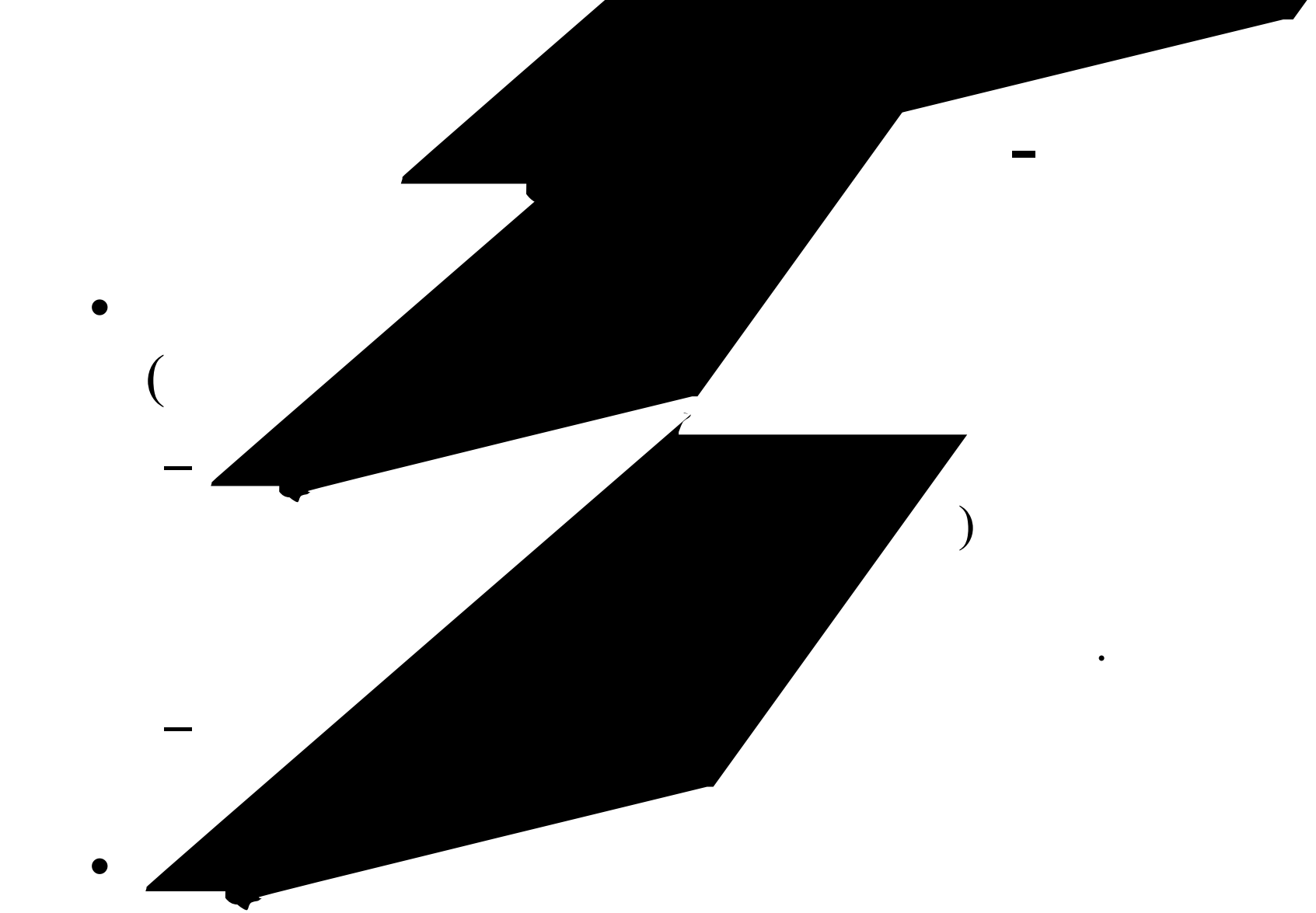
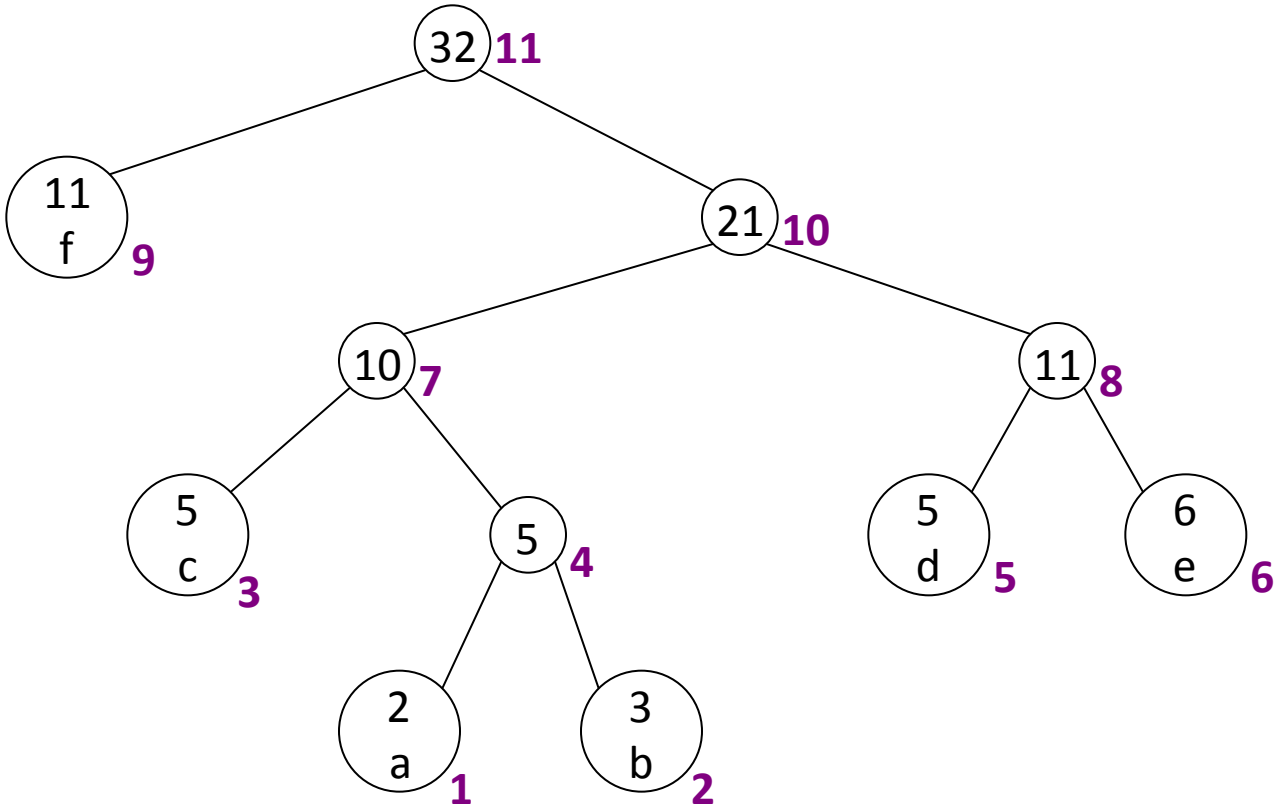
There is a contradiction?





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– *t*

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t+1

– *t*+1-

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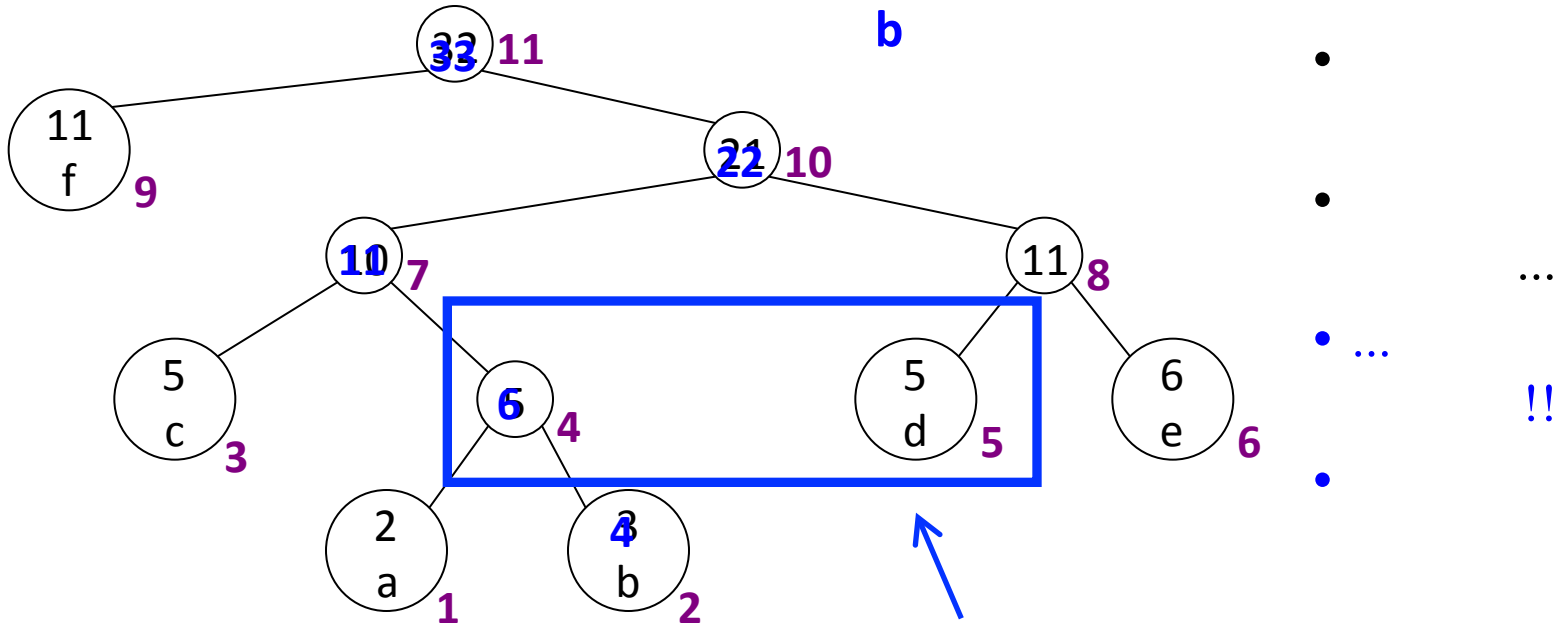
b
11
f
9
5
c
3
2
a
1
4
b
2
5
d
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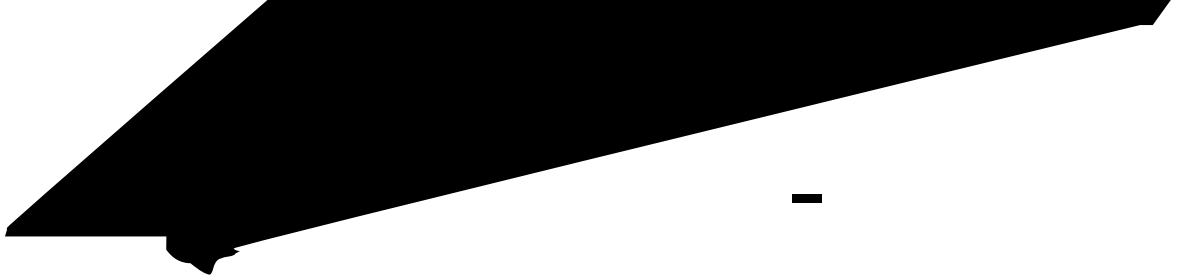
6
e
6
11
8
7
4
10
11
...
!!
no more ordered by
nondecreasing weight



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t ,

simple increment

process can be applied succesfully (How?)

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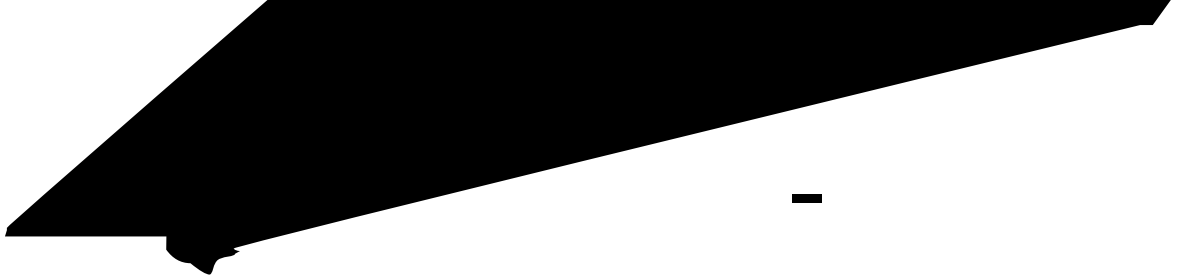

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- t+1-
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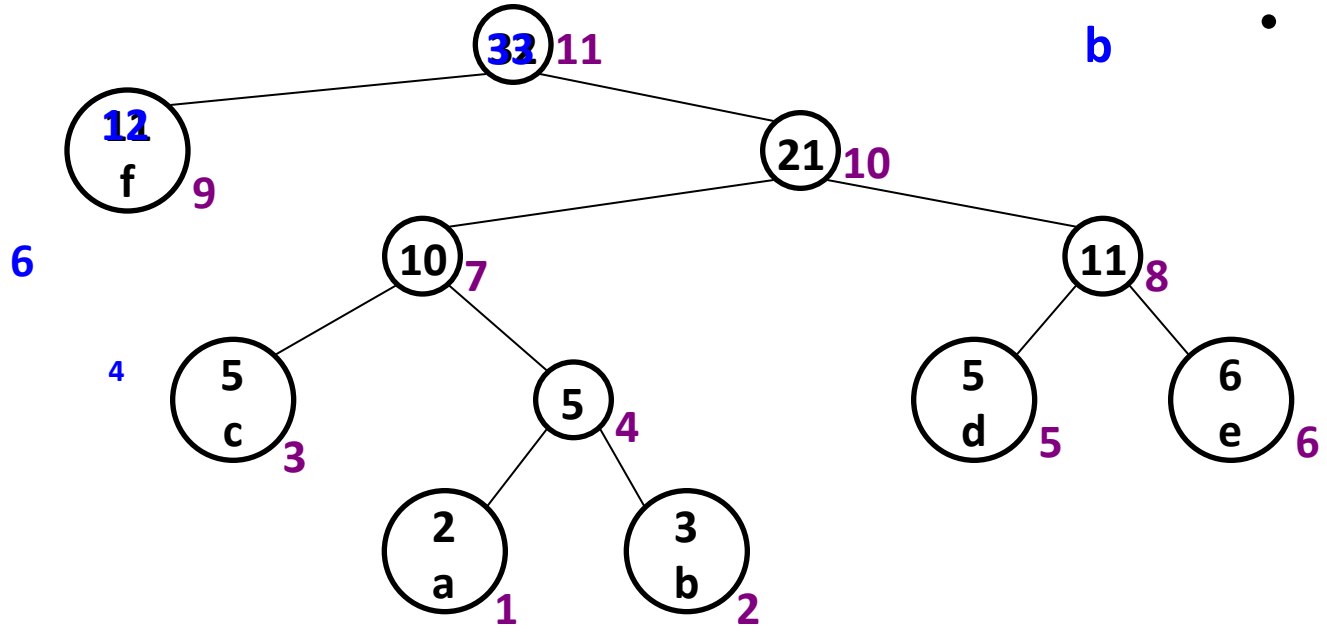

- **First phase**
  - 2:
  - 4:
  - 5
  - 8:
  - 9
  - :
- **Second phase**

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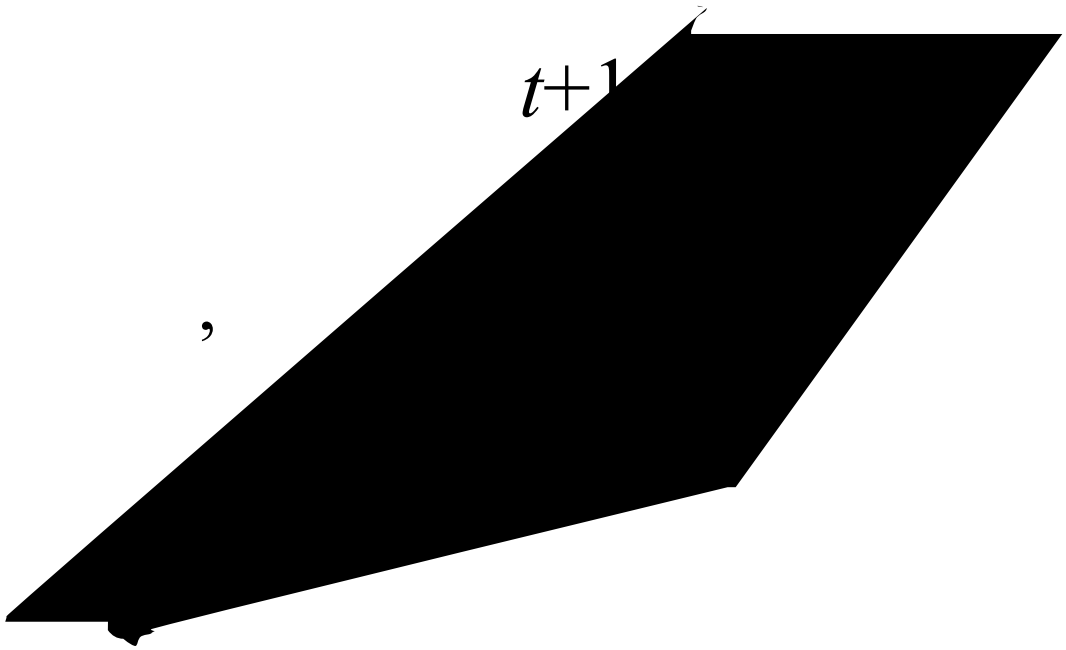
- 
t+1

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de for the 0-node

and some h ich is the new symbol

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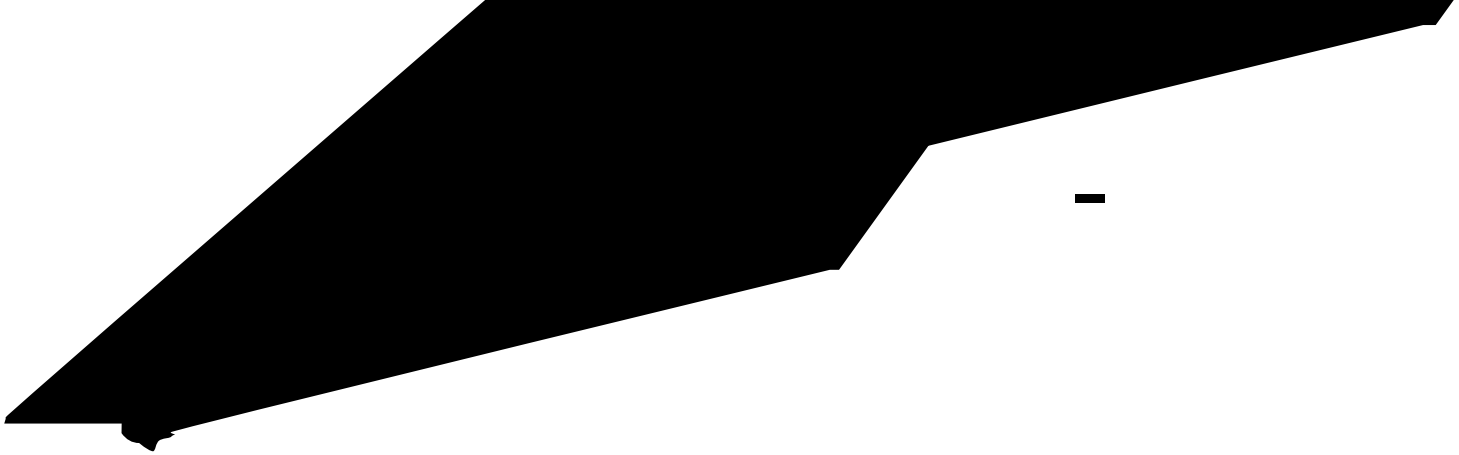

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(*compact*)

- **Exercise**
  -

e eae de eabe eae dcf

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- *T*=

t *n* ,

$$- \quad +1 \leq \quad \leq 2 \quad + \quad -4 \quad +2$$

S (1987)

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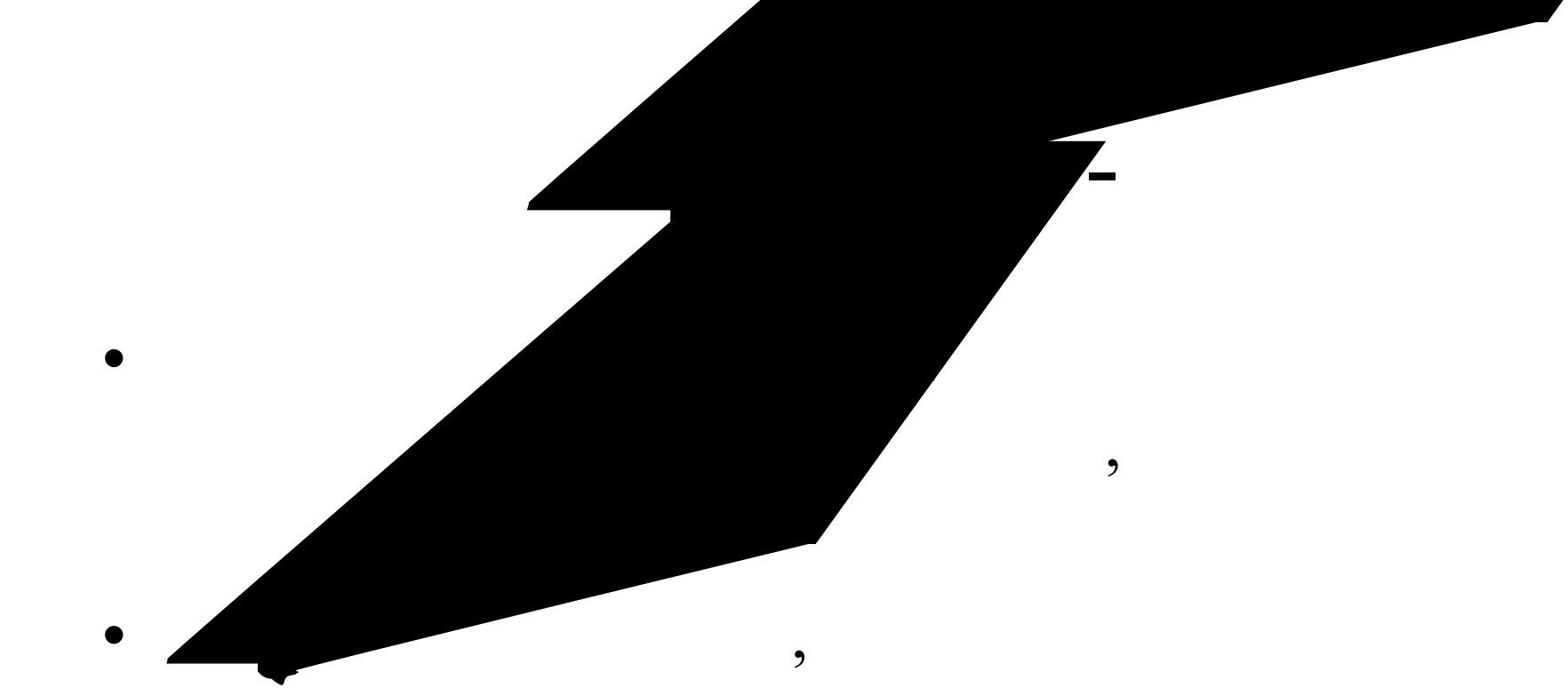

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(
) $\lceil d_t/2 \rceil$,
(
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– , 1

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- , $\sum_i w_i l_i$
, $\max_i l_i$. ,
, $\sum_i l_i$,

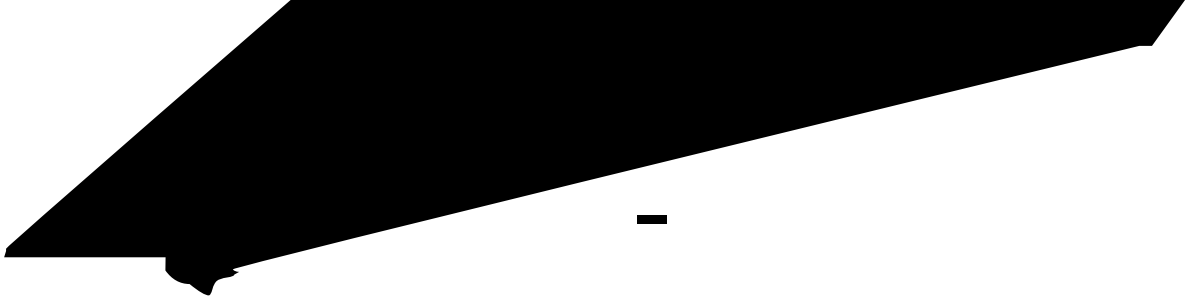
- , *implicit numbering*

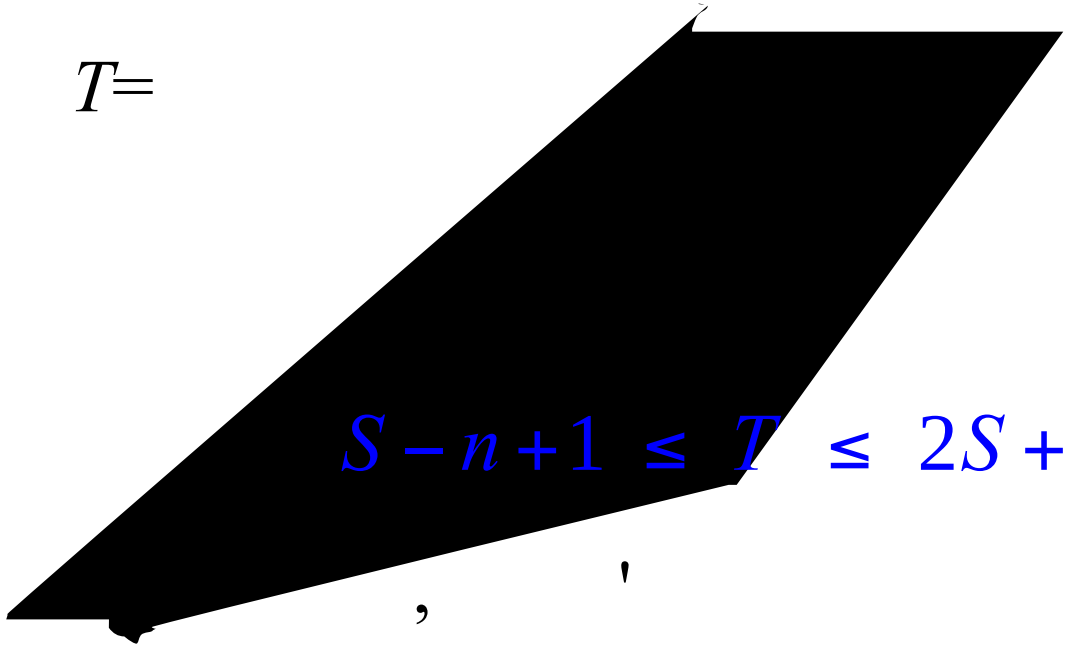
- ;
-
- (
),

- *invariant*

  – *for each weight w, all leaves of weight w precede (in the implicit numbering) all internal nodes of weight w*

- , , , ,



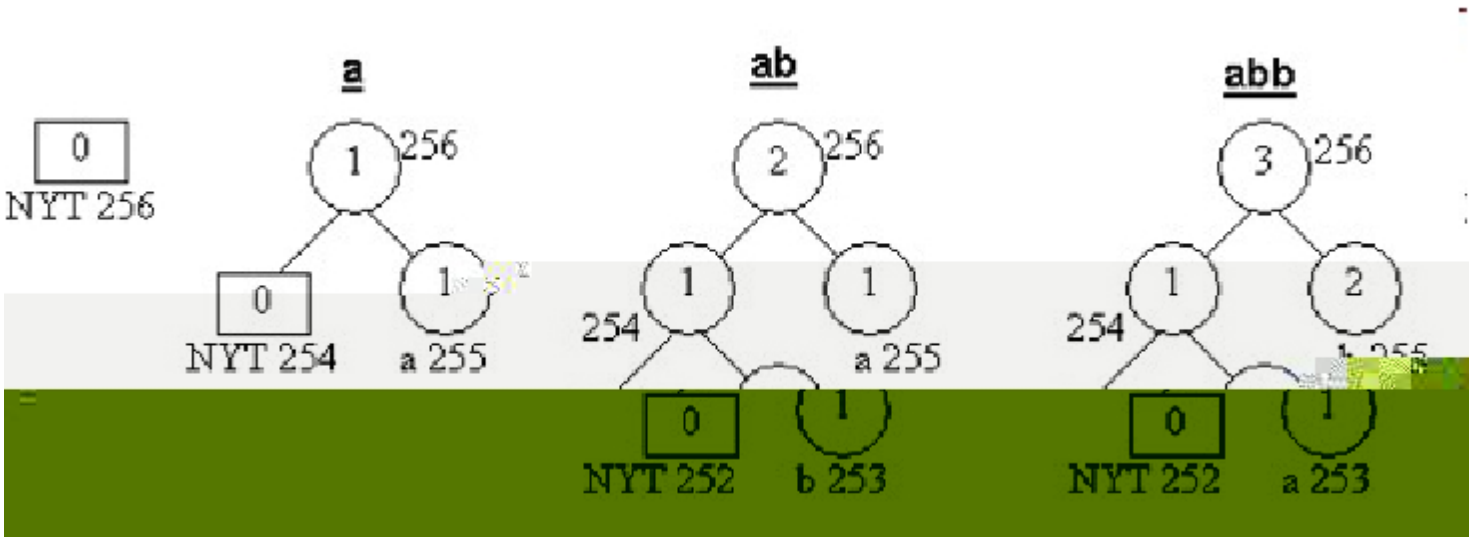
- $T=$  n ,

$$S - n + 1 \leq T \leq 2S + t - 2n + 1$$

- , '
- ,

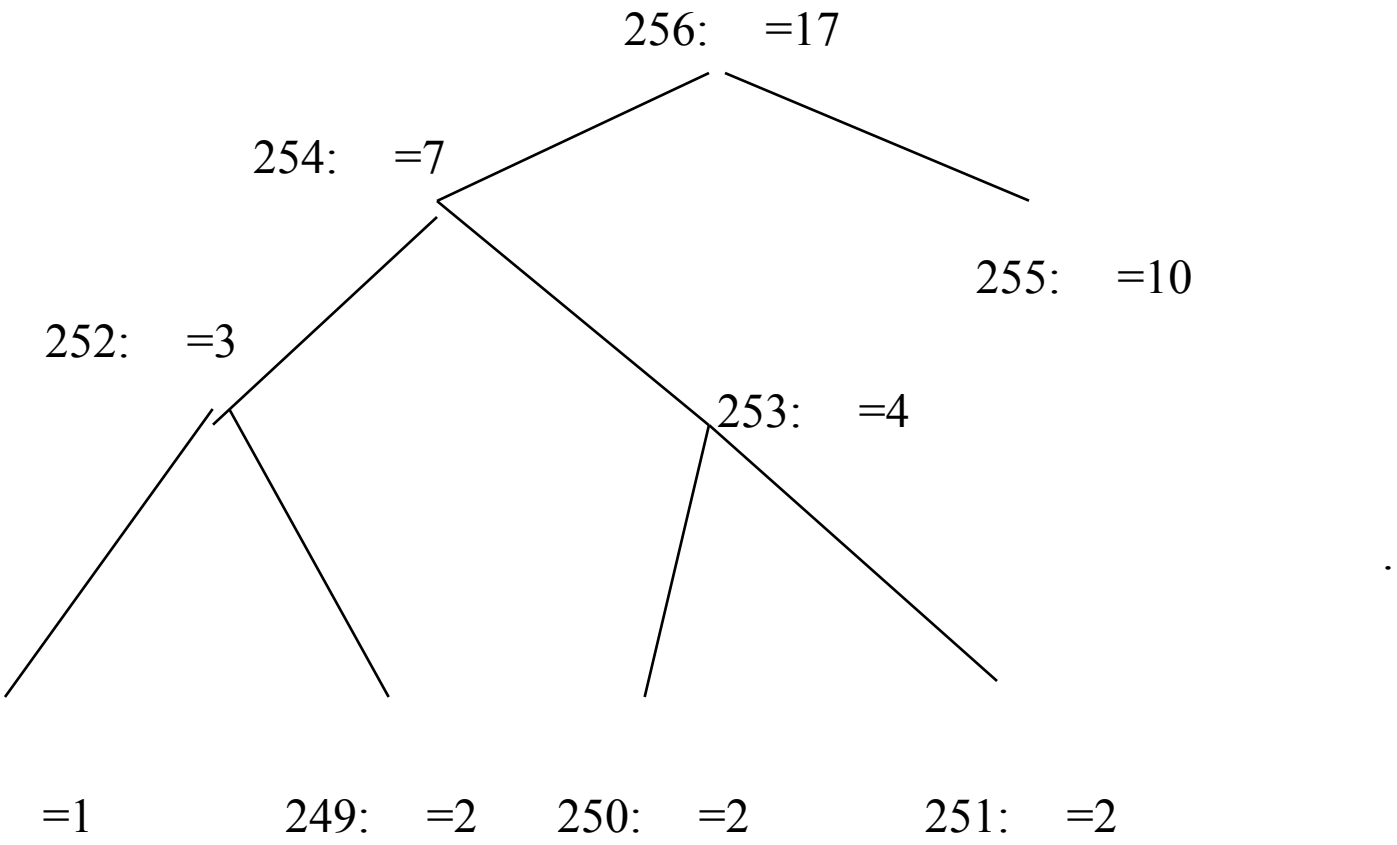
: 01100001011000100110001001100010011000100110001001100001

: 01100001**0**01100010**0111101**



: 01100001

: 01100010


256: =17
254: =7
255: =10
252: =3
253: =4
248: =1
249: =2
250: =2
251: =2

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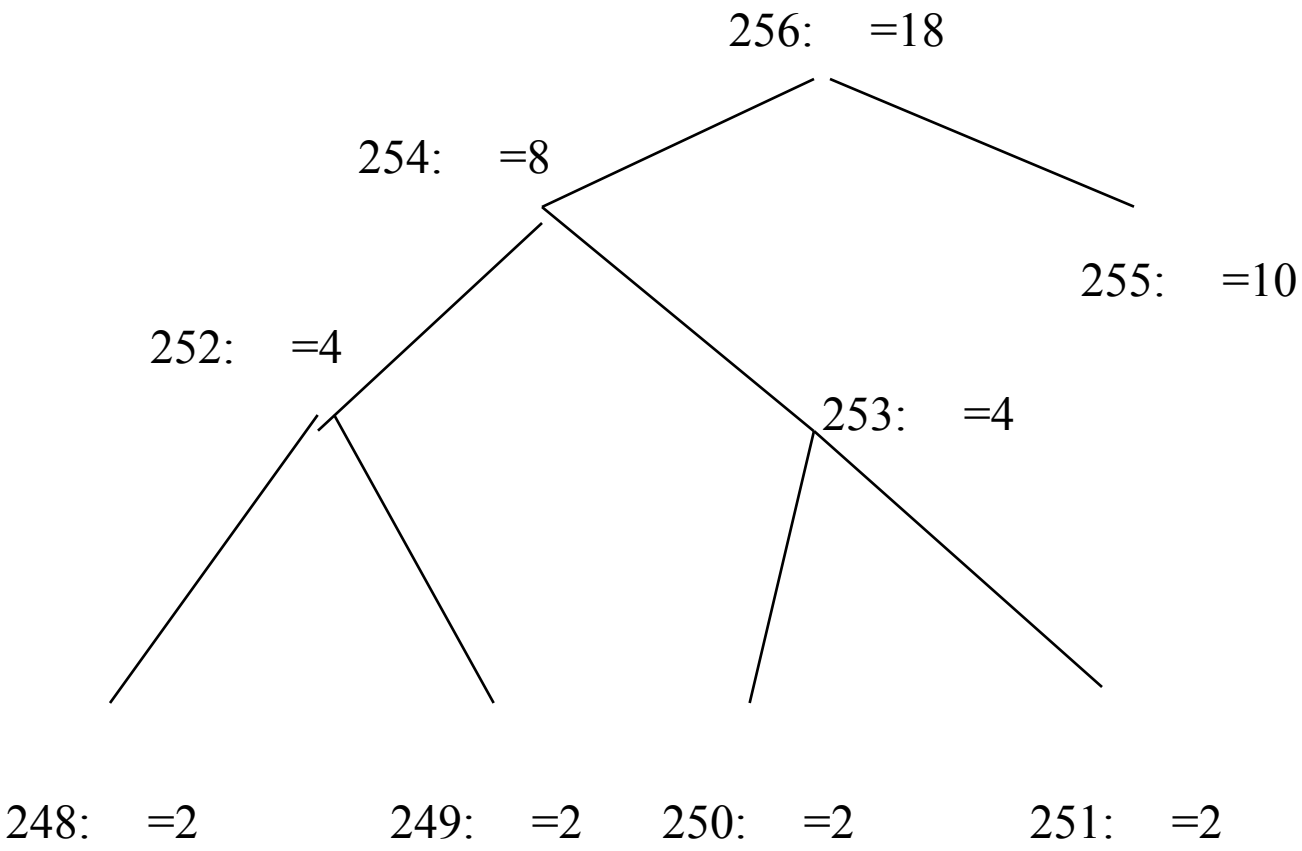
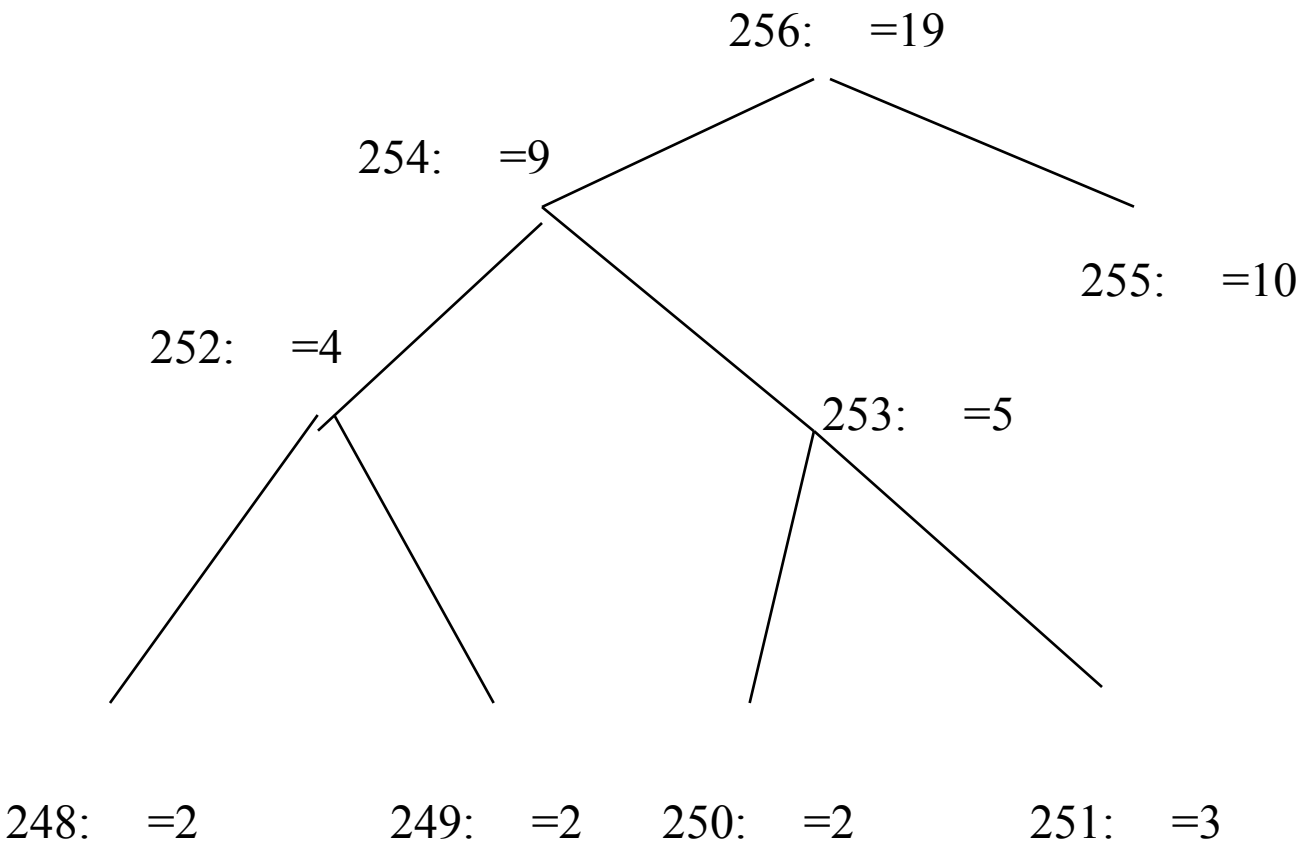
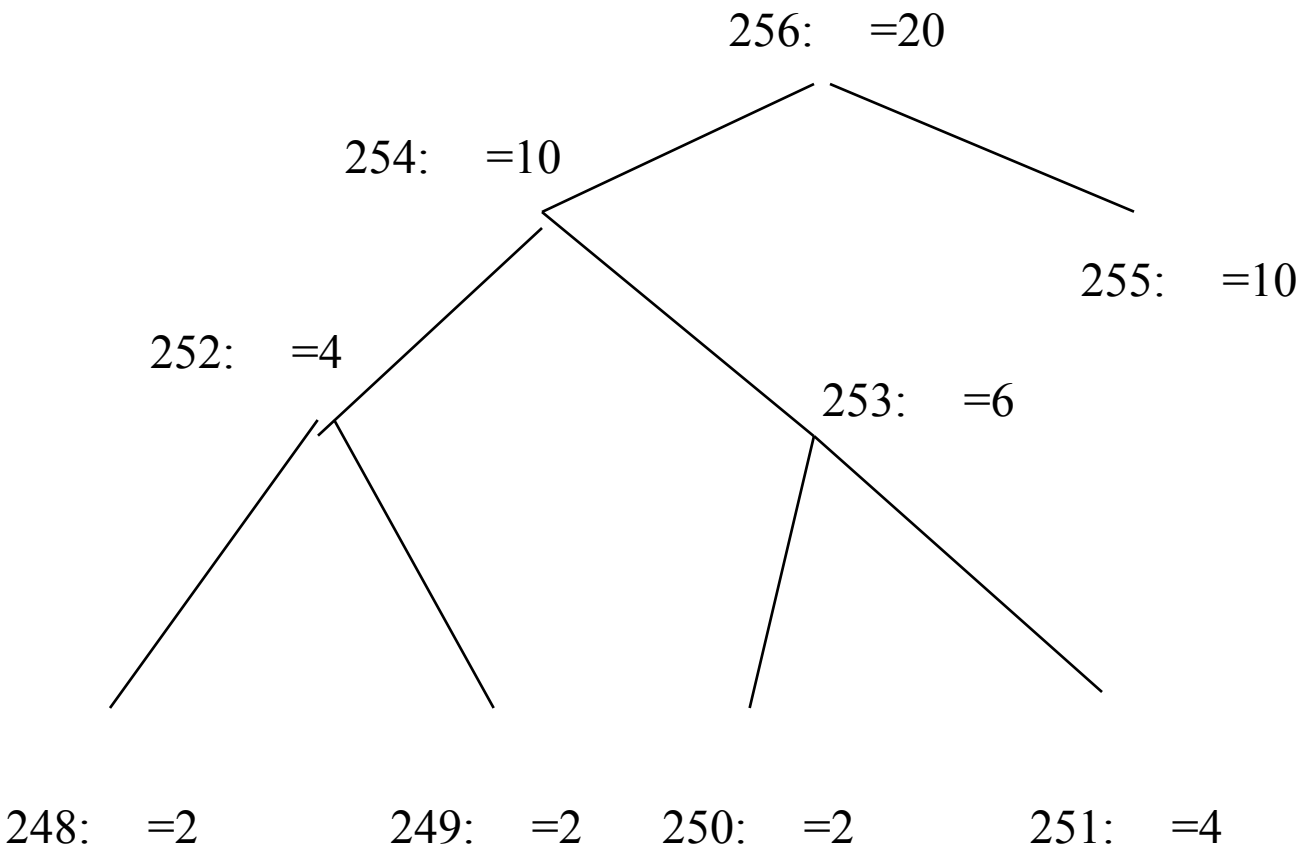
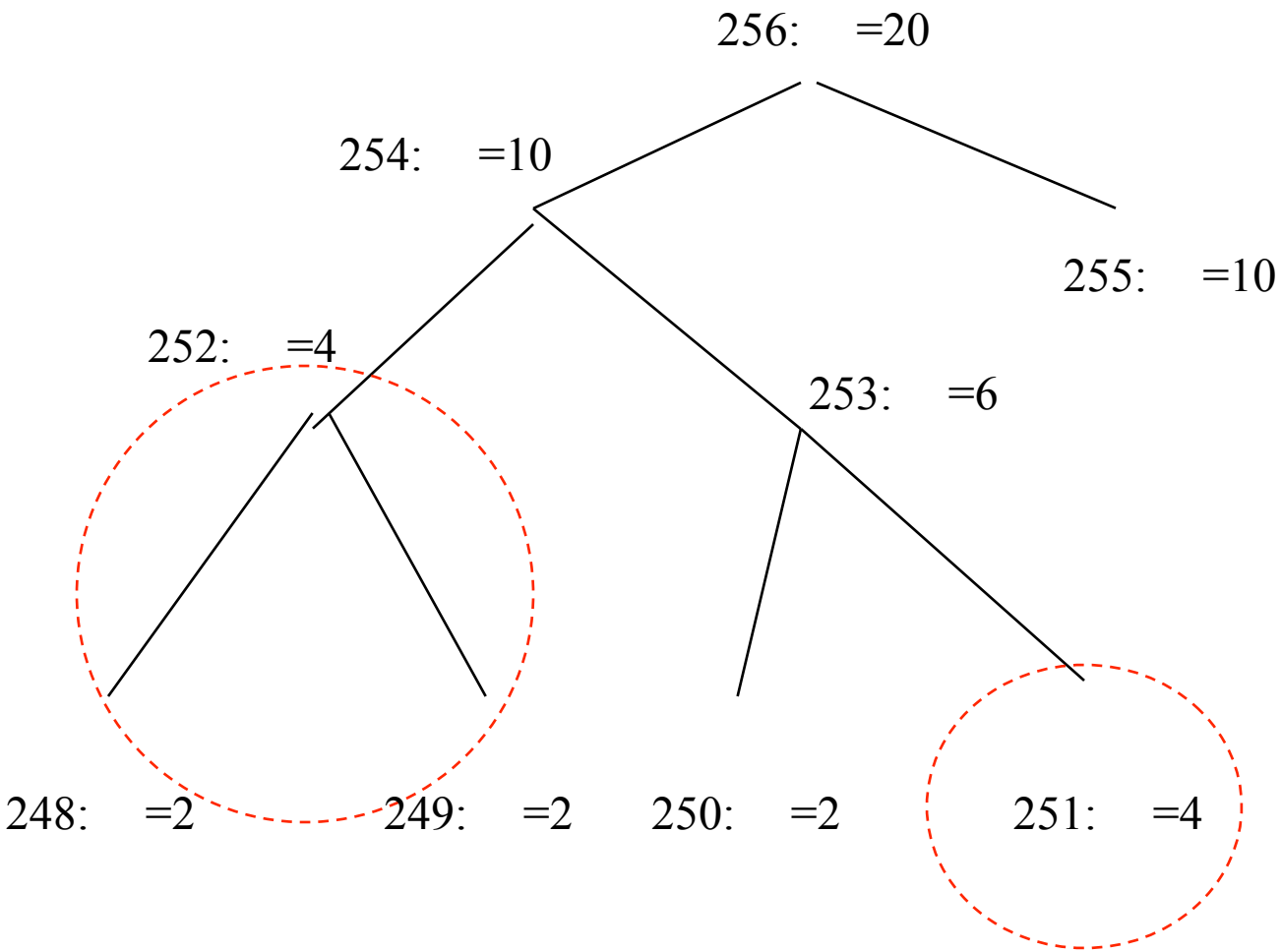
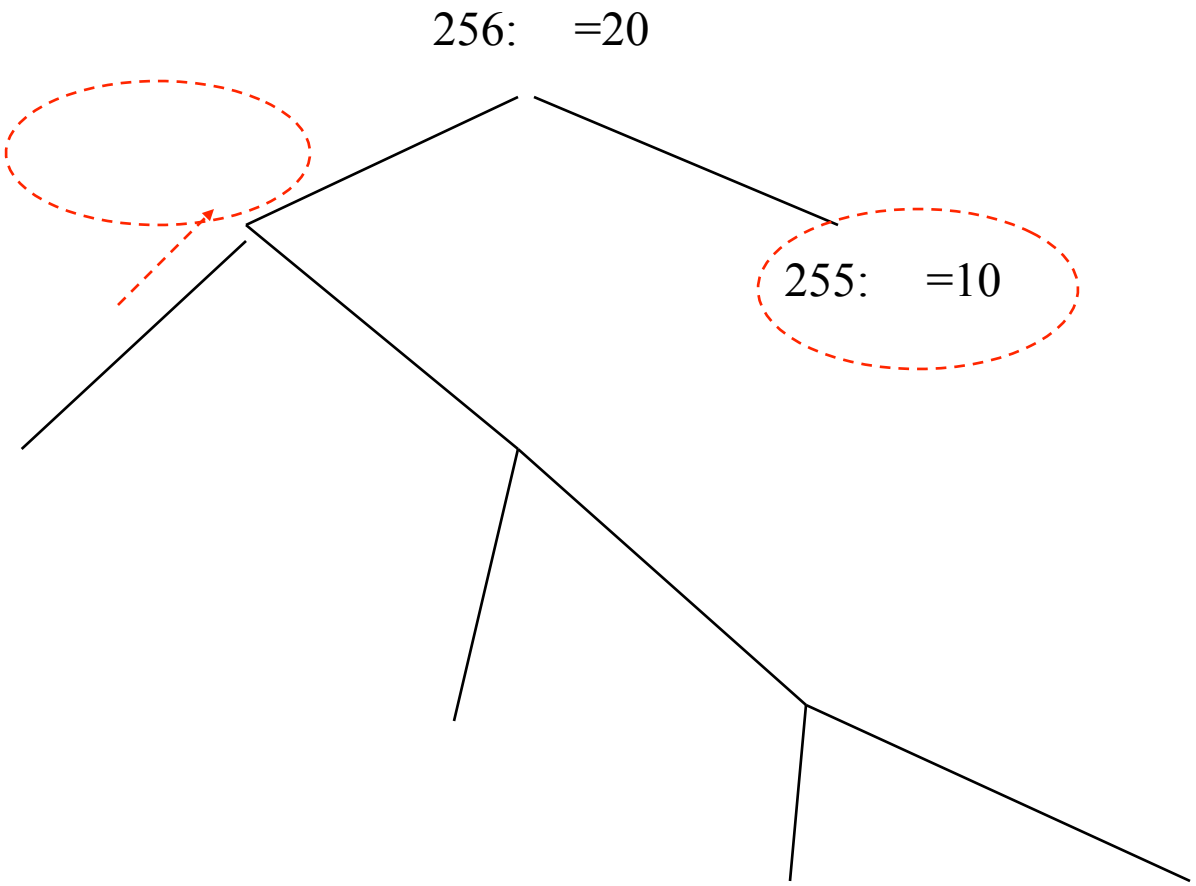
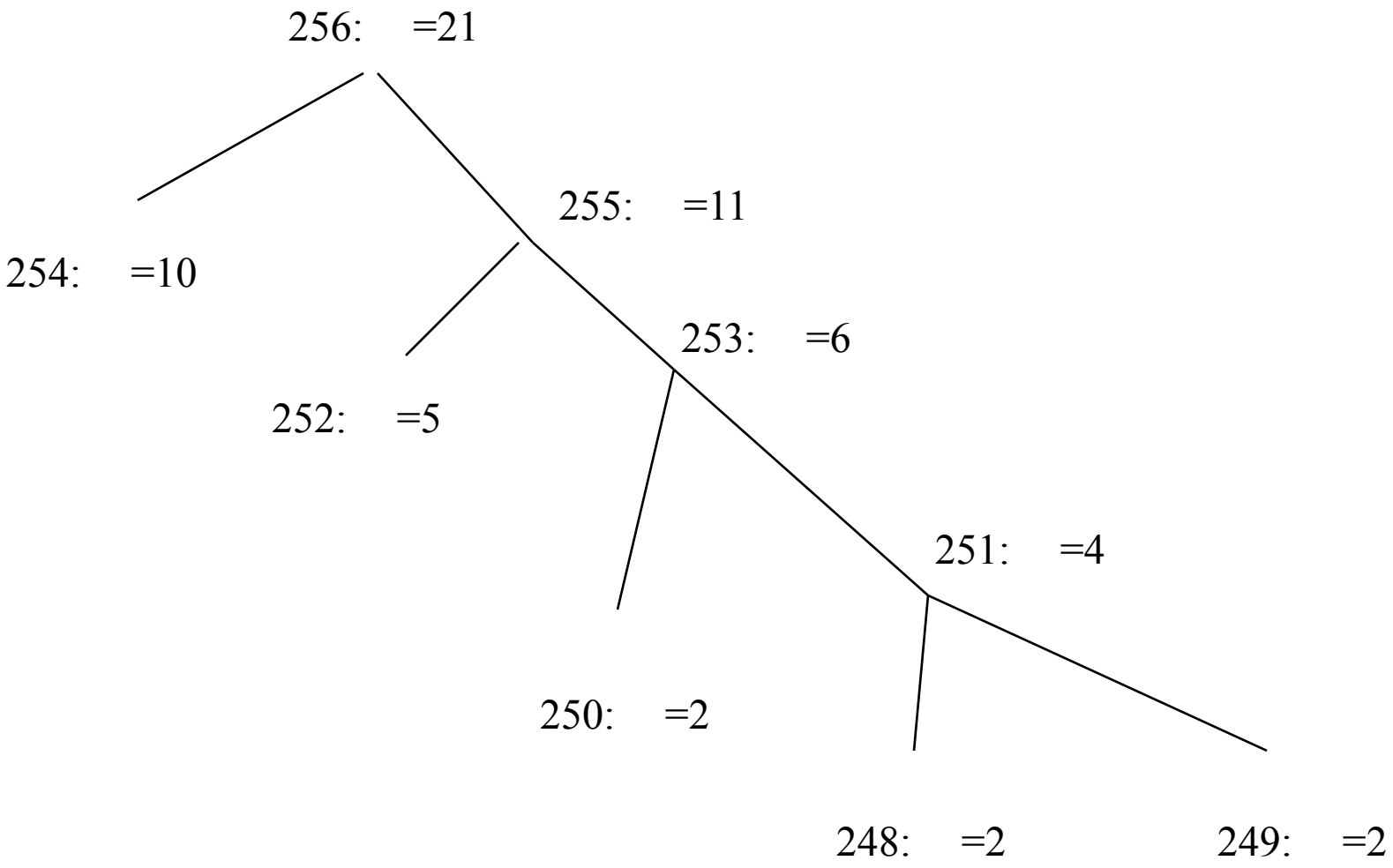

256: =18
254: =8
255: =10
252: =4
253: =4
248: =2
249: =2
250: =2
251: =2


256: =19
254: =9
255: =10
252: =4
253: =5
248: =2
249: =2
250: =2
251: =3


256: =20
254: =10
255: =10
252: =4
253: =6
248: =2
249: =2
250: =2
251: =4


256: =20
254: =10
255: =10
252: =4
253: =6
248: =2
249: =2
250: =2
251: =4


256: =20
255: =10


256: =21
255: =11
254: =10
253: =6
252: =5
251: =4
250: =2
248: =2
249: =2

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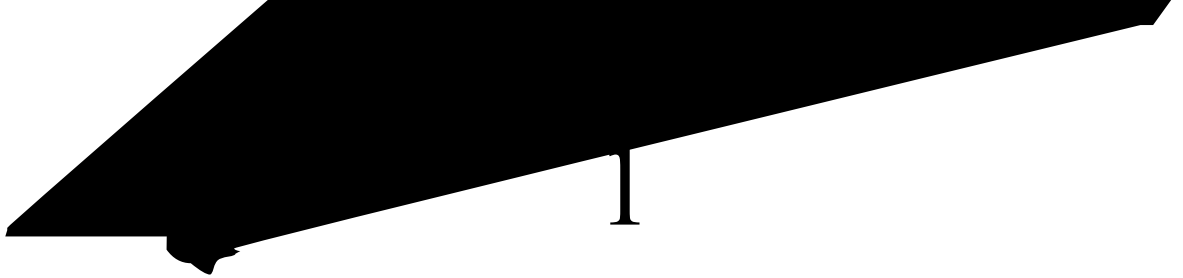

- **Size Determination of Huffman & LZW Encoded File**
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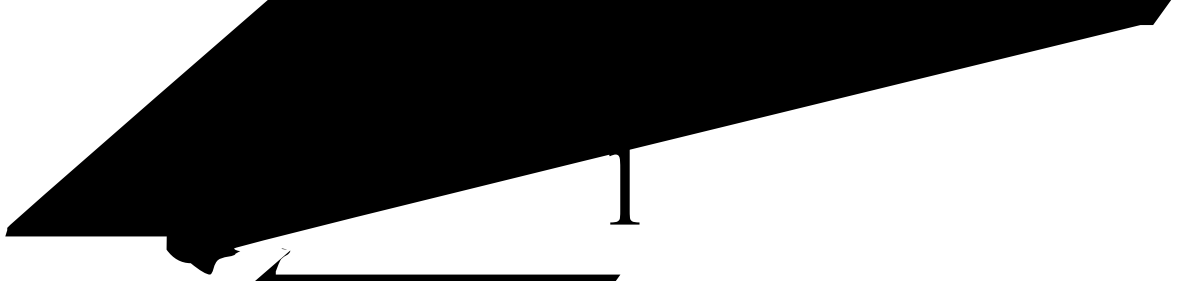

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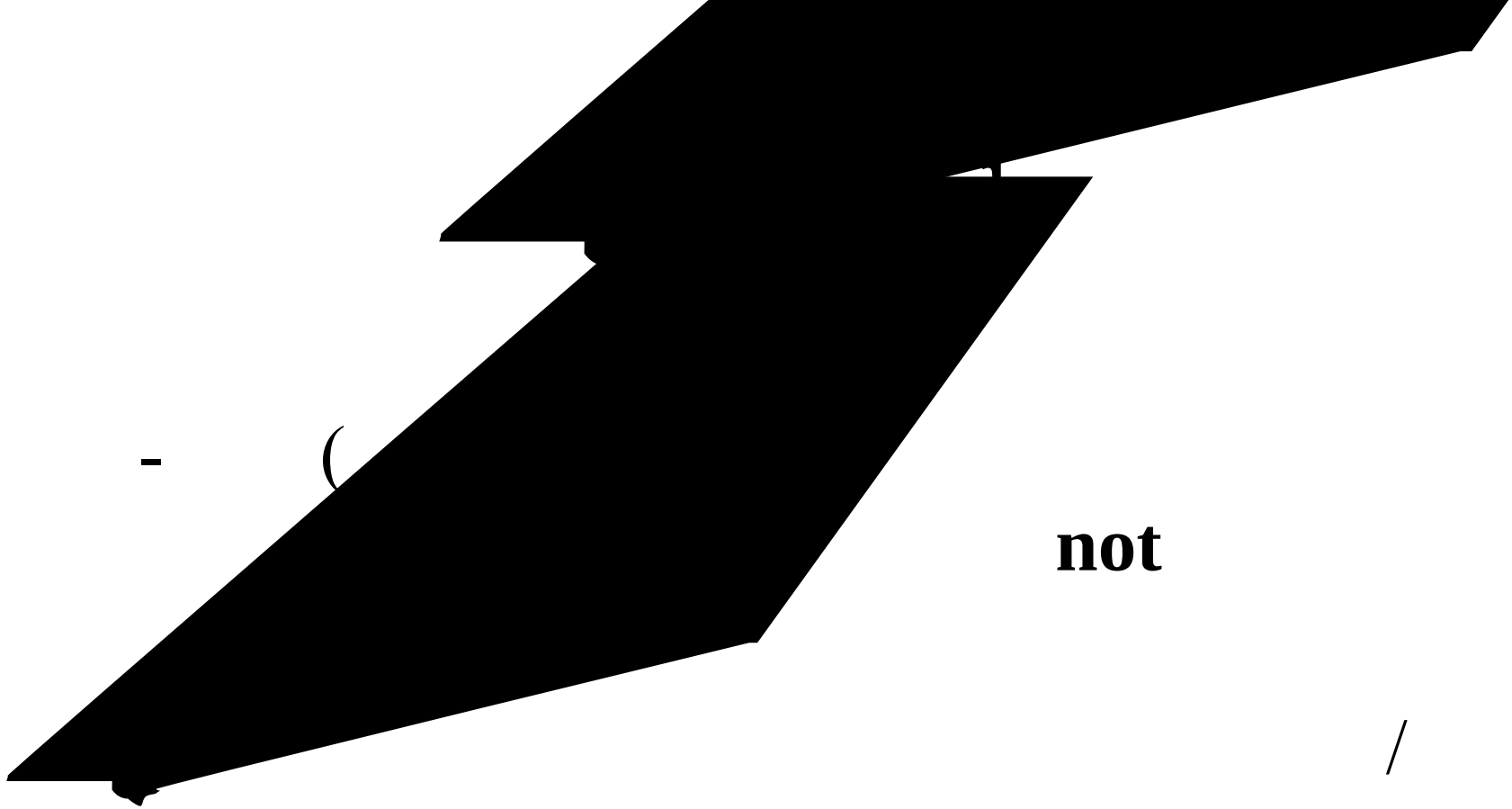

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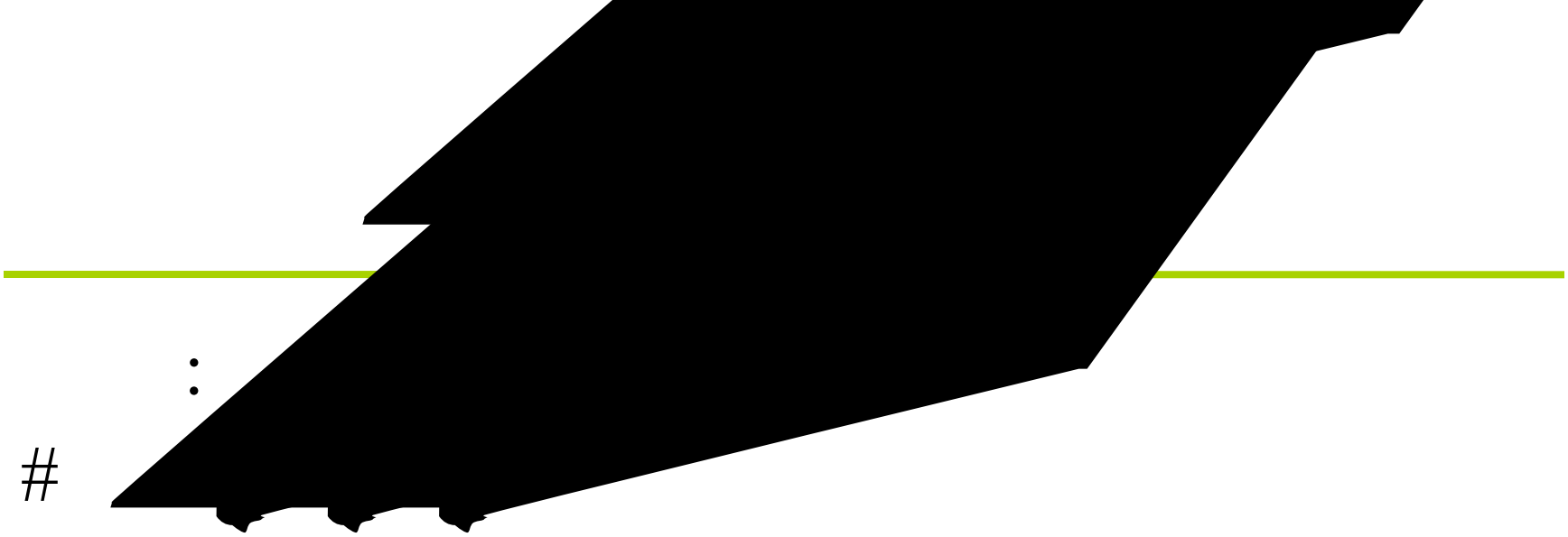
4 3 10 5 $



#BANANAS
S#BANANA
AS#BANAN
NAS#BANA
ANAS#BAN
NANAS#BA
ANANAS#B
BANANAS#

```
#BANANAS
ANANAS#B
ANAS#BAN
AS#BANAN
BANANAS#
NANAS#BA
NAS#BANA
S#BANANA
```

#BANANAS
ANANAS#B
ANAS#BAN
AS#BANAN
BANANAS#
NANAS#BA
NAS#BANA
S#BANANA

:

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$

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S

B

N

N

#

A

A

A

S
B
N
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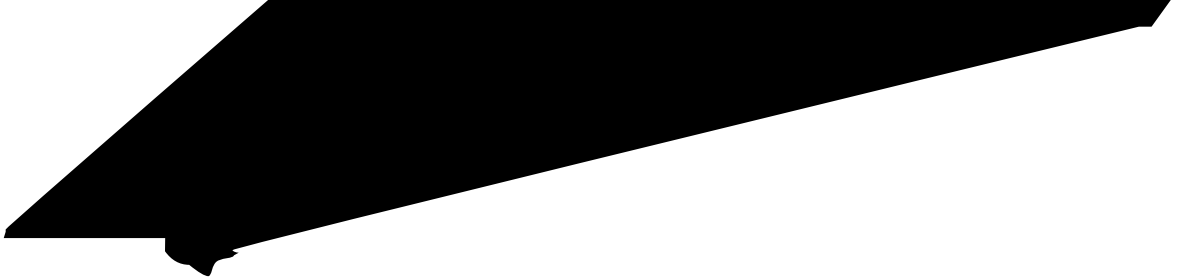
A
A
A

#AAABNNS



S#
BA
NA
NA
#B
AN
AN
AS

#B
AN
AN
AS
BA
NA
NA
S#

S#B
BAN
NAN
NAS
#BA
ANA
ANA
AS#

```
#BA
ANA
ANA
AS#
BAN
NAN
NAS
S#B
```

S#BA
BANA
NANA
NAS#
#BAN
ANAN
ANAS
AS#B

```
#BAN
ANAN
ANAS
AS#B
BANA
NANA
NAS#
S#BA
```

S#BAN
BANAN
NANAS
NAS#B
#BANA
ANANA
ANAS#
AS#BA

```
#BANA
ANANA
ANAS#
AS#BA
BANAN
NANAS
NAS#B
S#BAN
```

S#BANA
BANANA
NANAS#
NAS#BA
#BANAN
ANANAS
ANAS#B
AS#BAN

```
#BANAN
ANANAS
ANAS#B
AS#BAN
BANANA
NANAS#
NAS#BA
S#BANA
```

S#BANAN
BANANAS
NANAS#B
NAS#BAN
#BANANA
ANANAS#
ANAS#BA
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```
#BANANA
ANANAS#
ANAS#BA
AS#BANA
BANANAS
NANAS#B
NAS#BAN
S#BANAN
```

S#BANANA
BANANAS#
NANAS#BA
NAS#BANA
#BANANAS
ANANAS#B
ANAS#BAN
AS#BANAN

(?)

#BANANAS
ANANAS#B
ANAS#BAN
AS#BANAN
BANANAS#
NANAS#BA
NAS#BANA
S#BANANA

( )

function ( )

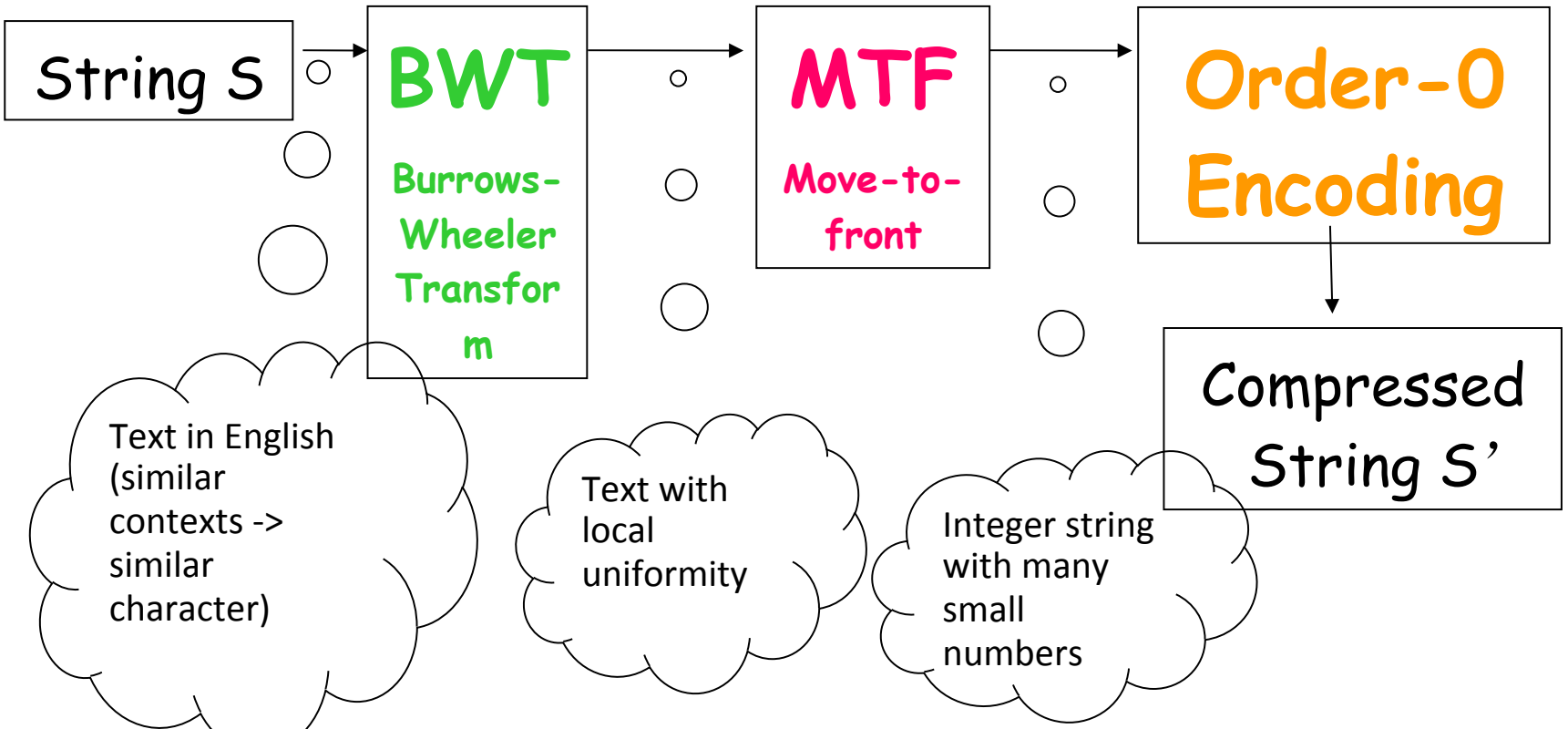
repeat ( ) **times**

//

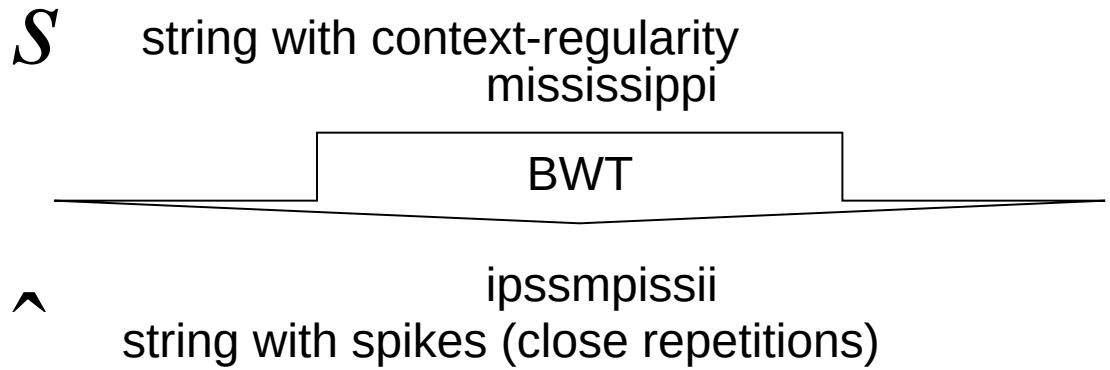
return (' ')

BW0

The Main Burrows-Wheeler Compression Algorithm:

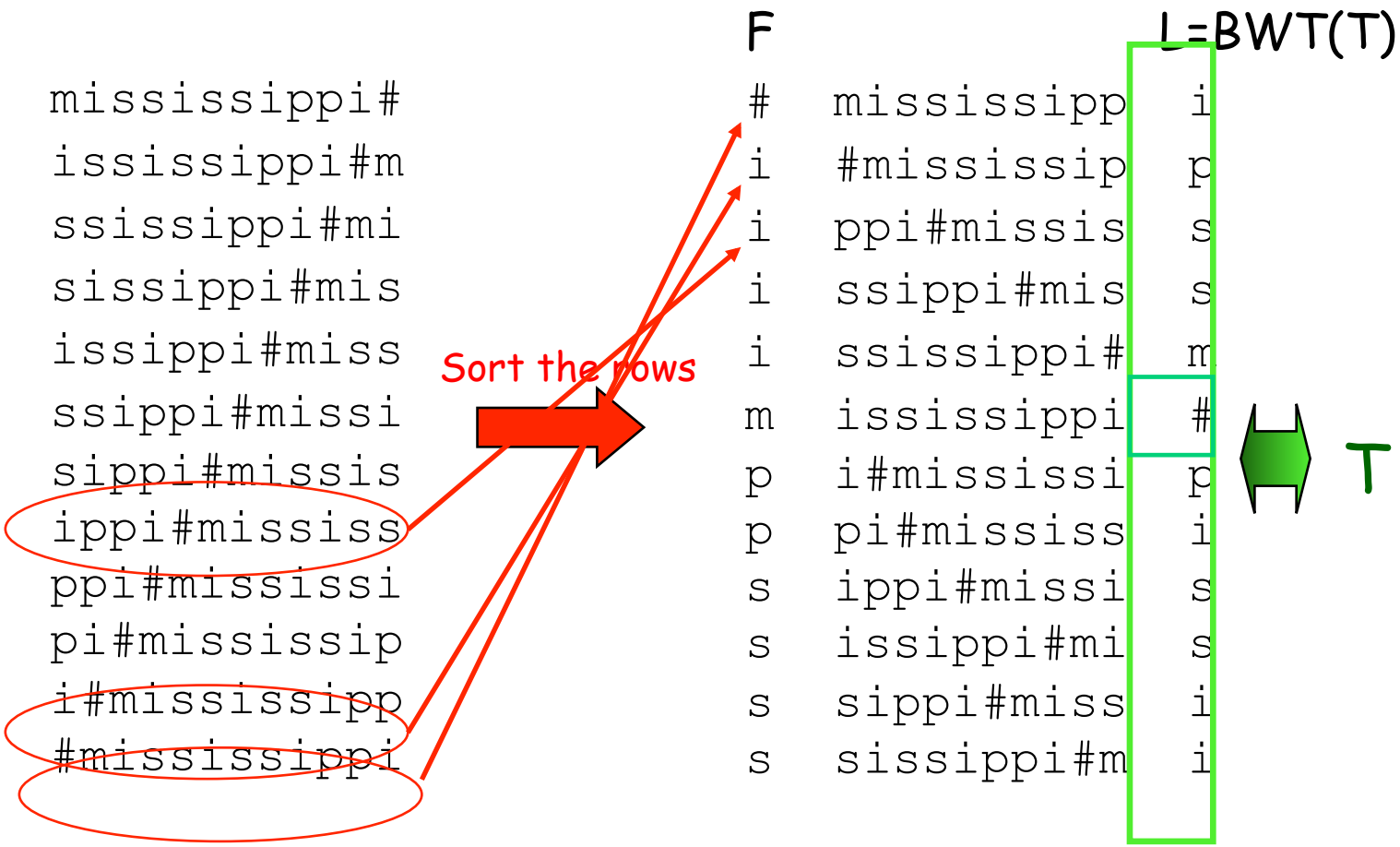


The BWT

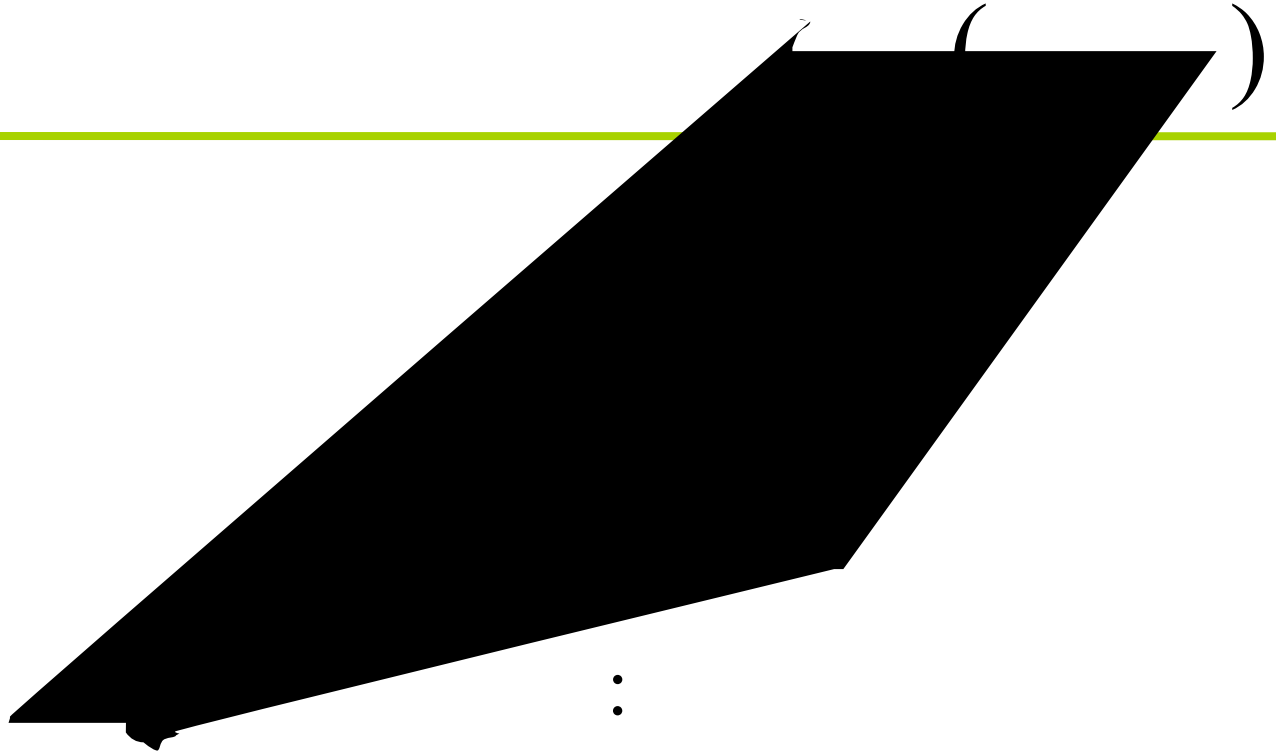

S
string with context-regularity
mississippi
BWT
ipssmpissii
$\hat{}$
string with spikes (close repetitions)

The BWT

T = mississippi#



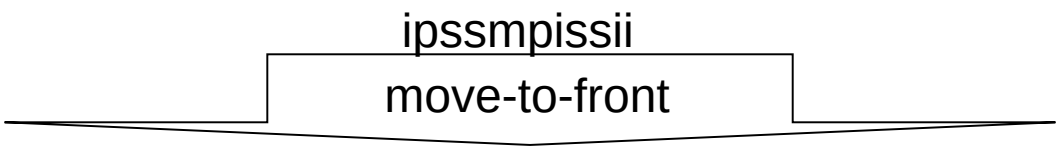
BWT sorts the characters by their post-context



Move To Front

- By Bentley, Sleator, Tarjan and Wei（’86)

$\hat{}$ string with spikes (close repetitions)



0,0,0,0,0,2,4,3,0,1,0

S' integer string with small numbers

Move to Front

abracadabra		**a**,b,r,c,d

Move to Front

abracadabra		**a**,b,r,c,d
a**b**racadabra	0	a,**b**,r,c,d

Move to Front

abracadabra		**a**,b,r,c,d
a**b**racadabra	0	a,**b**,r,c,d
ab**r**acadabra	0,1	b,a,**r**,c,d

Move to Front

abracadabra		**a**,b,r,c,d
a**b**racadabra	0	a,**b**,r,c,d
ab**r**acadabra	0,1	b,a,**r**,c,d
abr**a**cadabra	0,1,2	r,b,**a**,c,d

Move to Front

abr**a**cadabra	0,1,2	r,b,**a**,c,d
abra**c**adabra	0,1,2,2	a,r,b,**c**,d

Move to Front

abracadabra		**a**,b,r,c,d
a**b**racadabra	0	a,**b**,r,c,d
ab**r**acadabra	0,1	b,a,**r**,c,d
abr**a**cadabra	0,1,2	r,b,**a**,c,d
abra**c**adabra	0,1,2,2	a,r,b,**c**,d
abrac**a**dabra	0,1,2,2,3	c,**a**,r,b,d

Move to Front

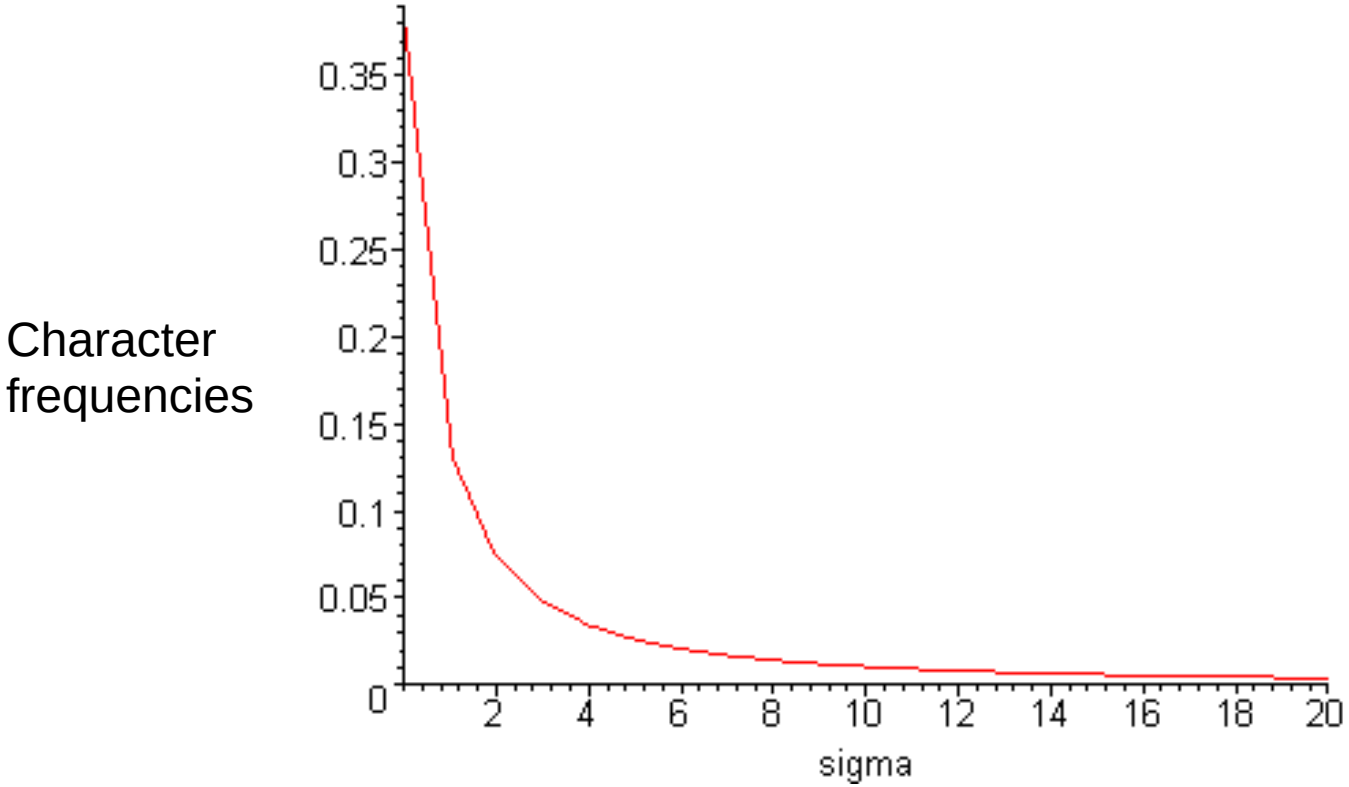
abracadabra		**a**,b,r,c,d
a**b**racadabra	0	a,**b**,r,c,d
ab**r**acadabra	0,1	b,a,**r**,c,d
abr**a**cadabra	0,1,2	r,b,**a**,c,d
abra**c**adabra	0,1,2,2	a,r,b,**c**,d
abrac**a**dabra	0,1,2,2,3	c,**a**,r,b,d
abraca**d**abra	0,1,2,2,3,1	a,c,r,b,**d**

Move to Front

abracadabra		**a**,b,r,c,d
a**b**racadabra	0	a,**b**,r,c,d
ab**r**acadabra	0,1	b,a,**r**,c,d
abr**a**cadabra	0,1,2	r,b,**a**,c,d
abra**c**adabra	0,1,2,2	a,r,b,**c**,d
abrac**a**dabra	0,1,2,2,3	c,**a**,r,b,d
abraca**d**abra	0,1,2,2,3,1	a,c,r,b,**d**
abracadabra	0,1,2,2,3,1,4,1,4,4,2	

After MTF

- Now we have a string with small numbers: lots of 0s, many 1s, …
- Skewed frequencies: Run Arithmetic!



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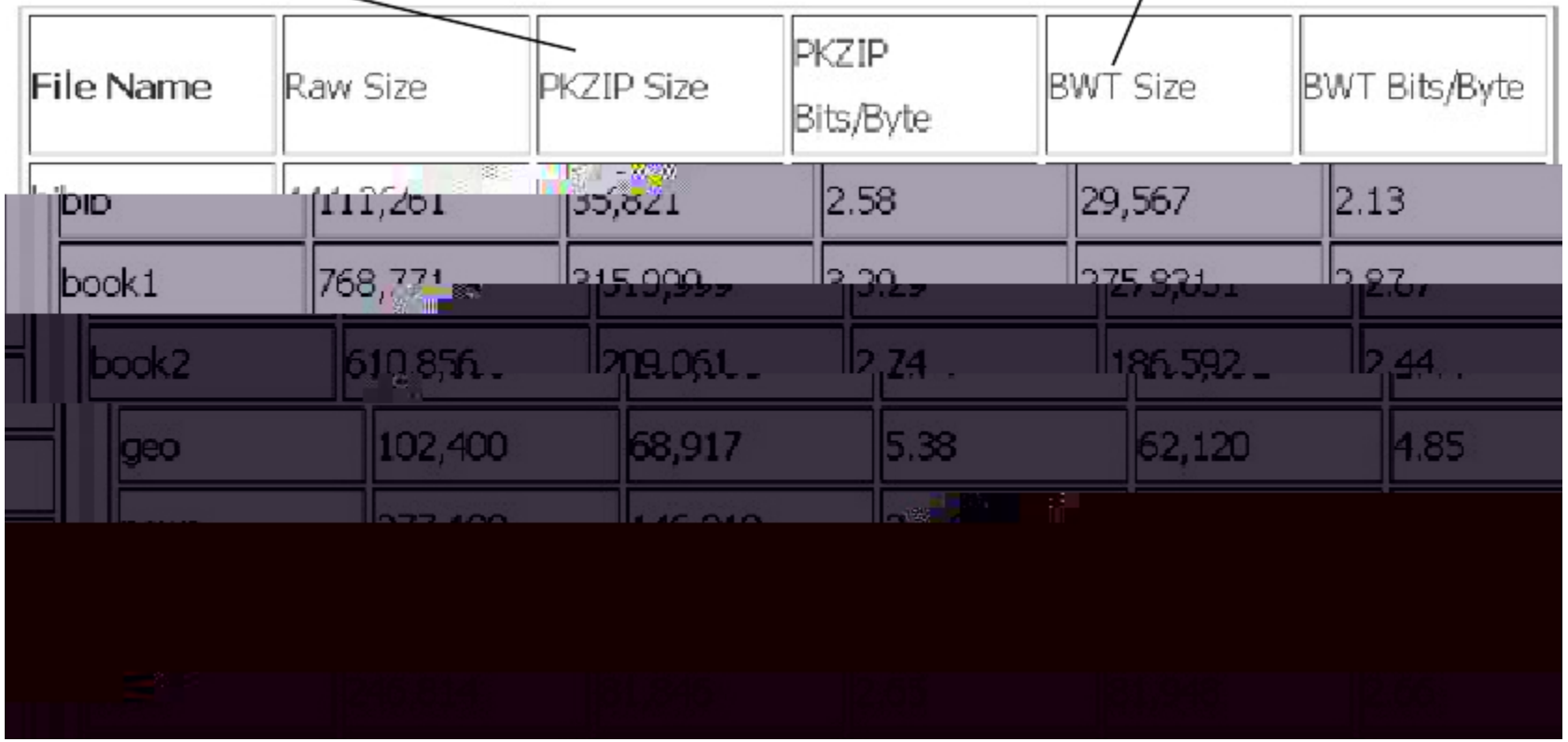

File Name	Raw Size	PKZIP Size	PKZIP Bits/Byte	BWT Size	BWT Bits/Byte
bib	111,261	35,821	2.58	29,567	2.13
book1	768,771	315,999	3.29	275,831	2.87
book2	610,856	209,061	2.74	186,592	2.44
geo	102,400	68,917	5.38	62,120	4.85

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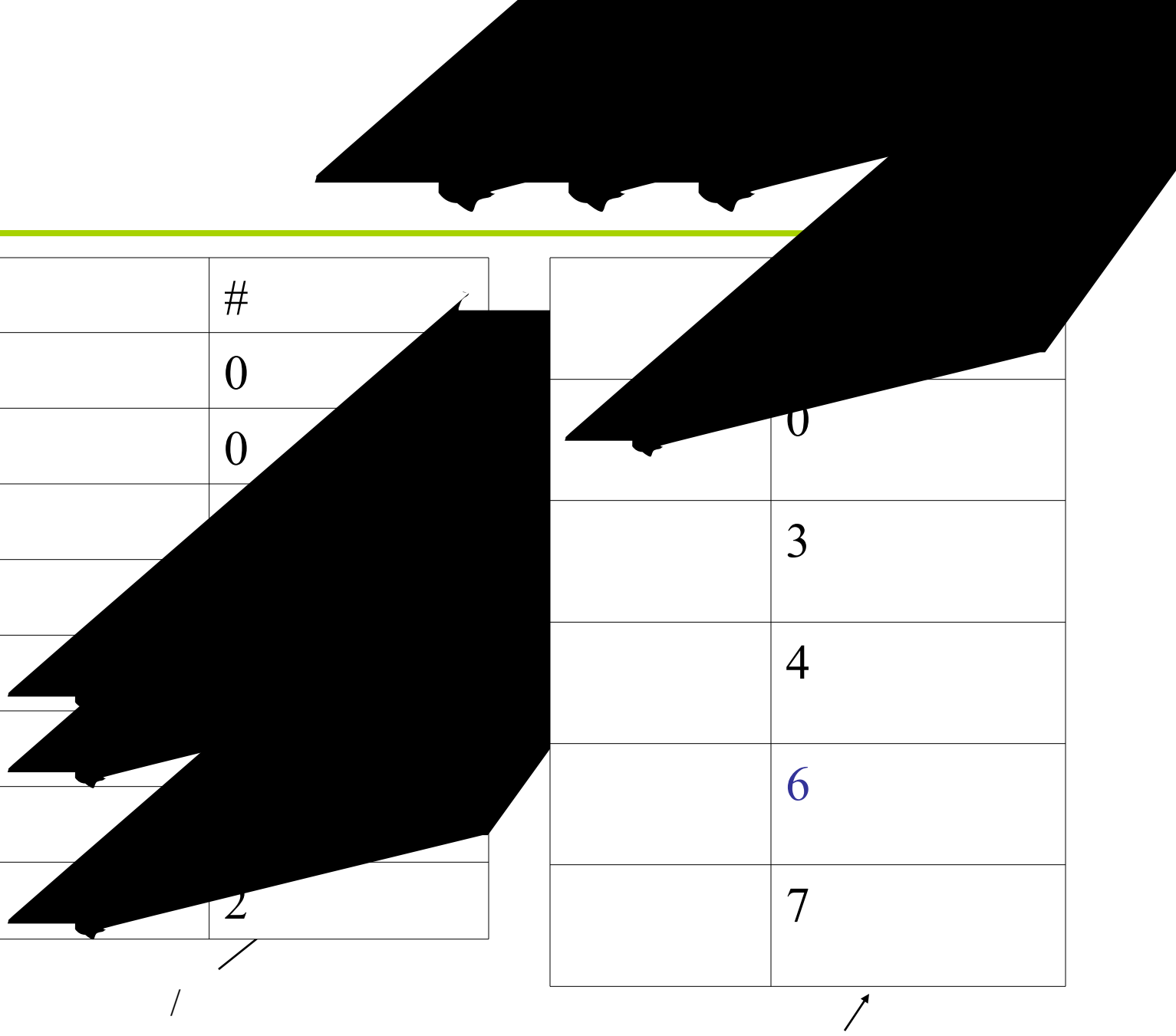

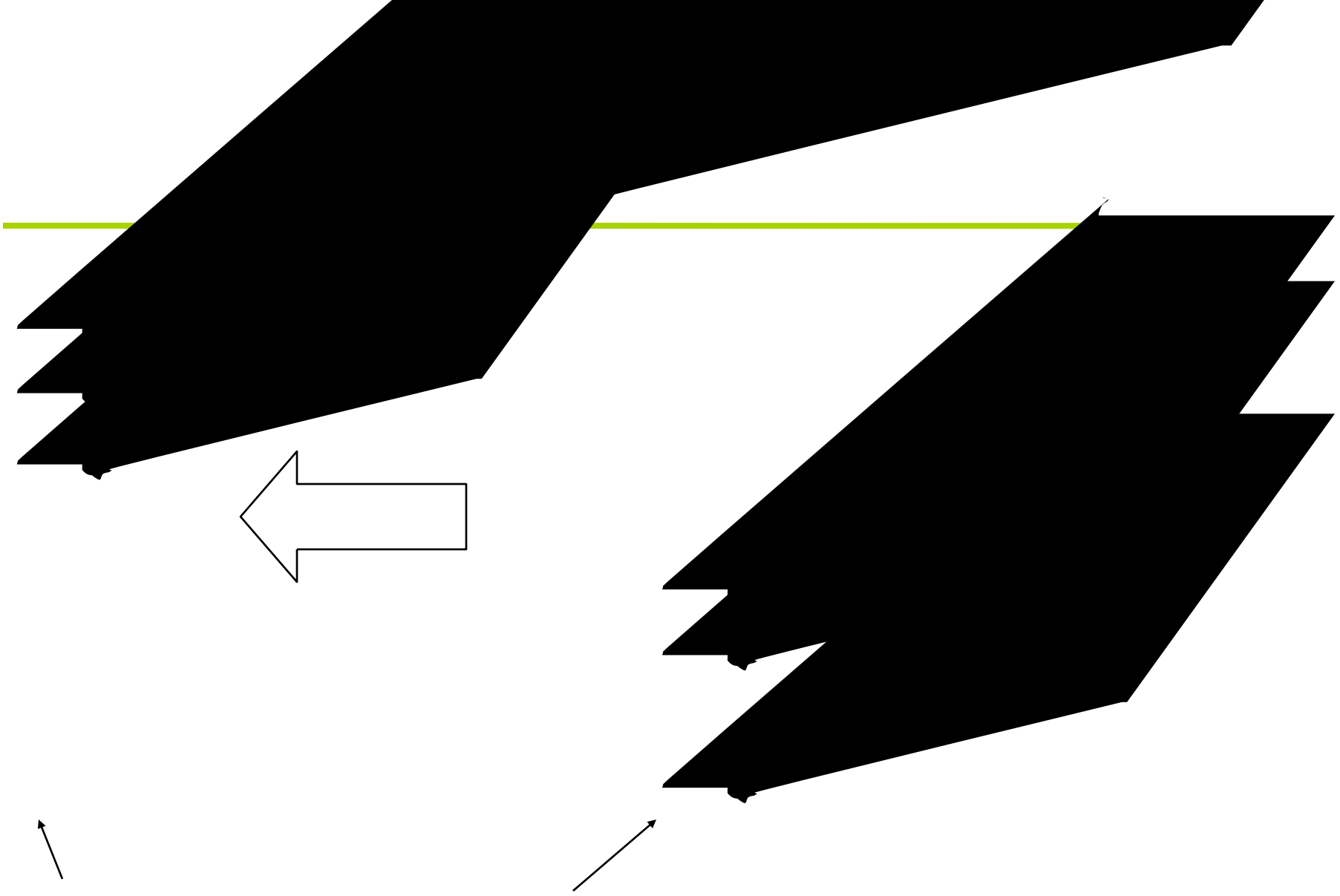

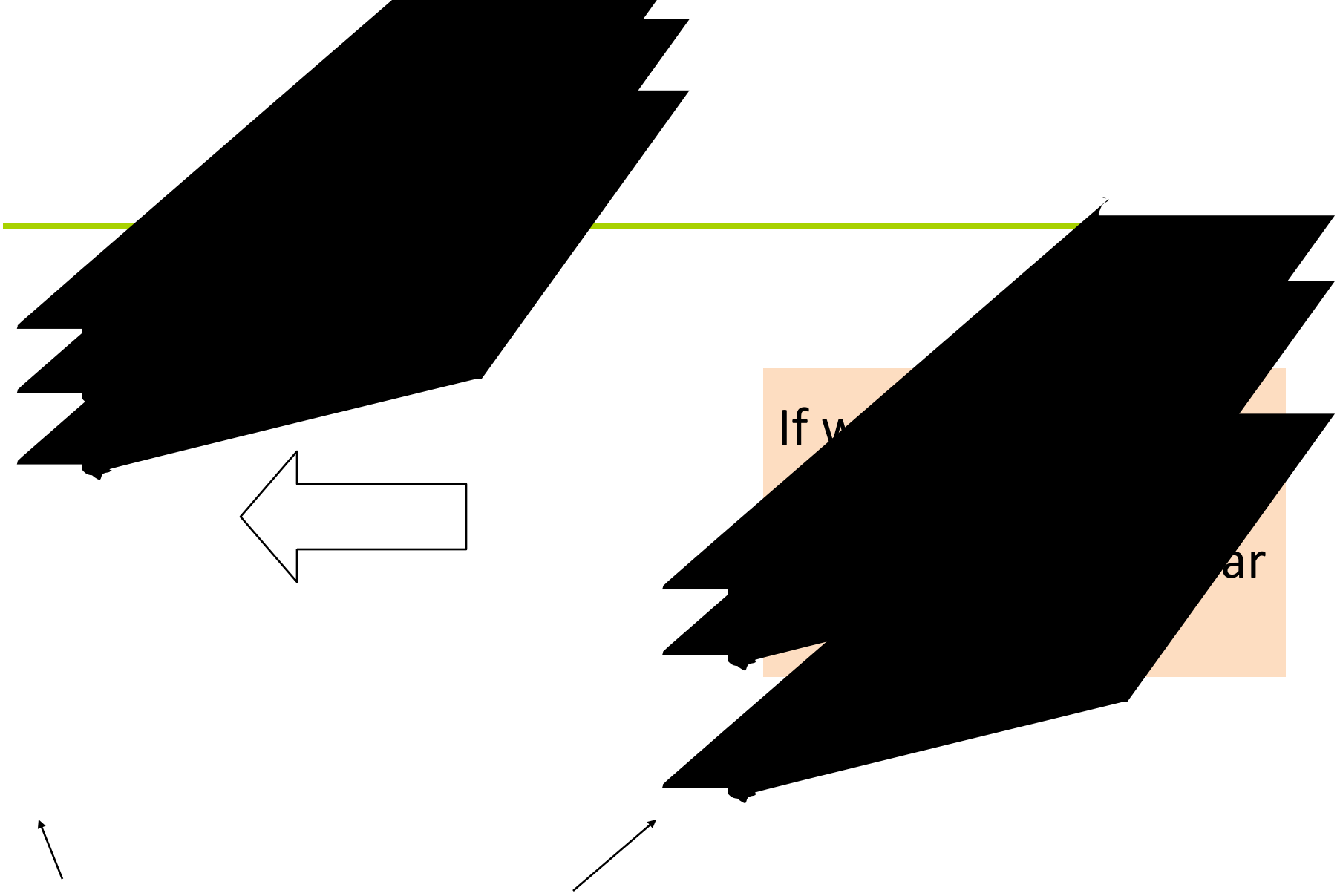
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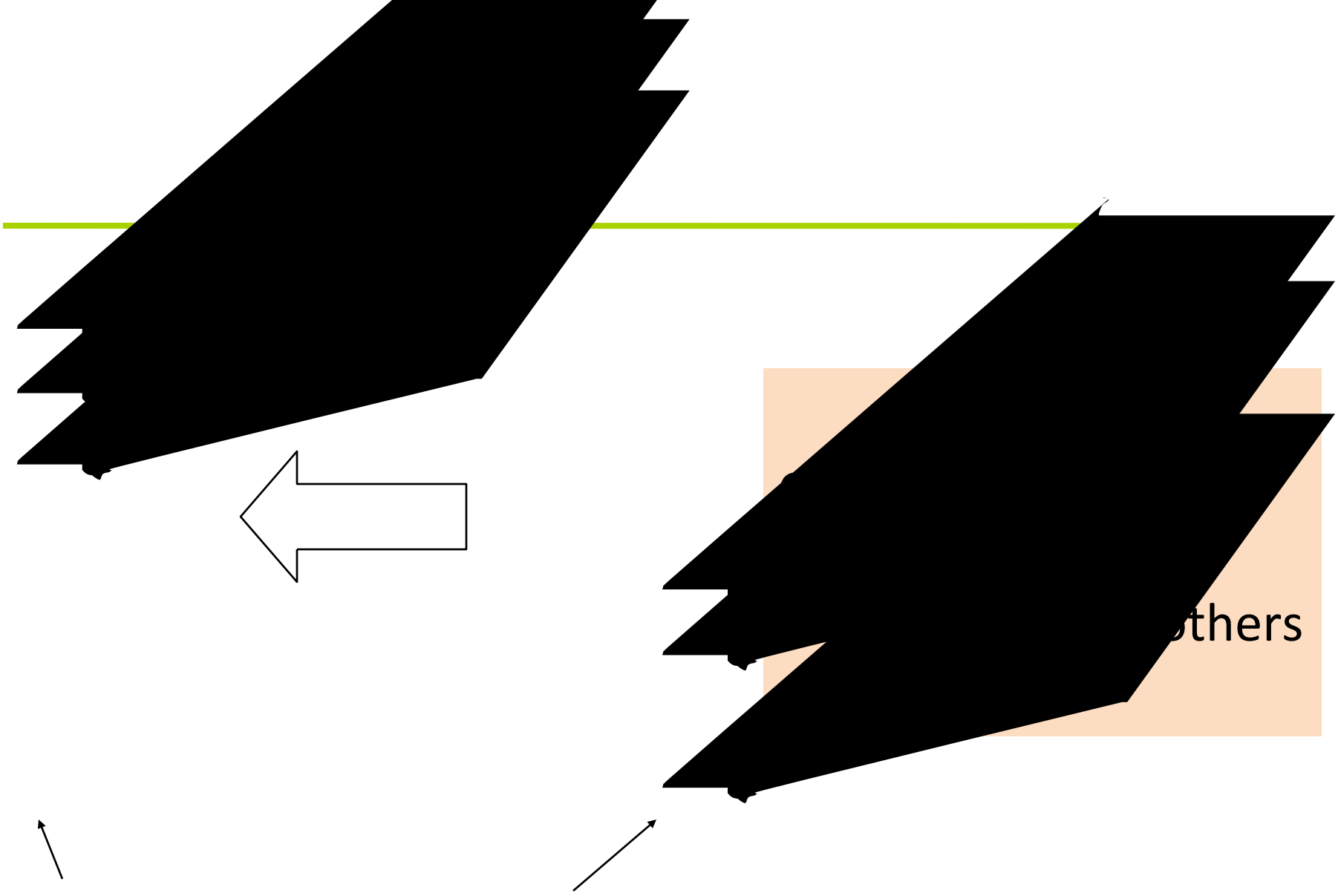
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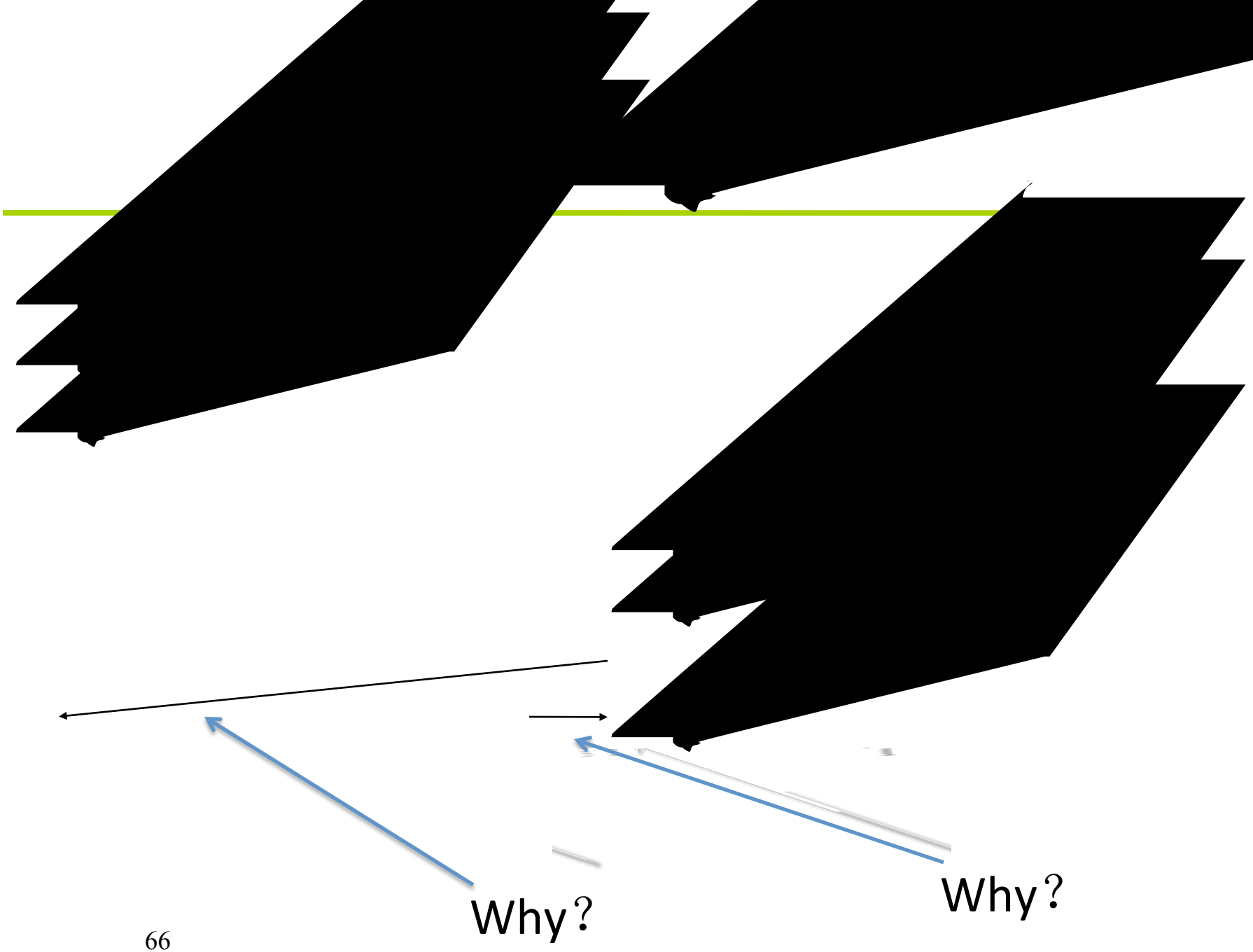
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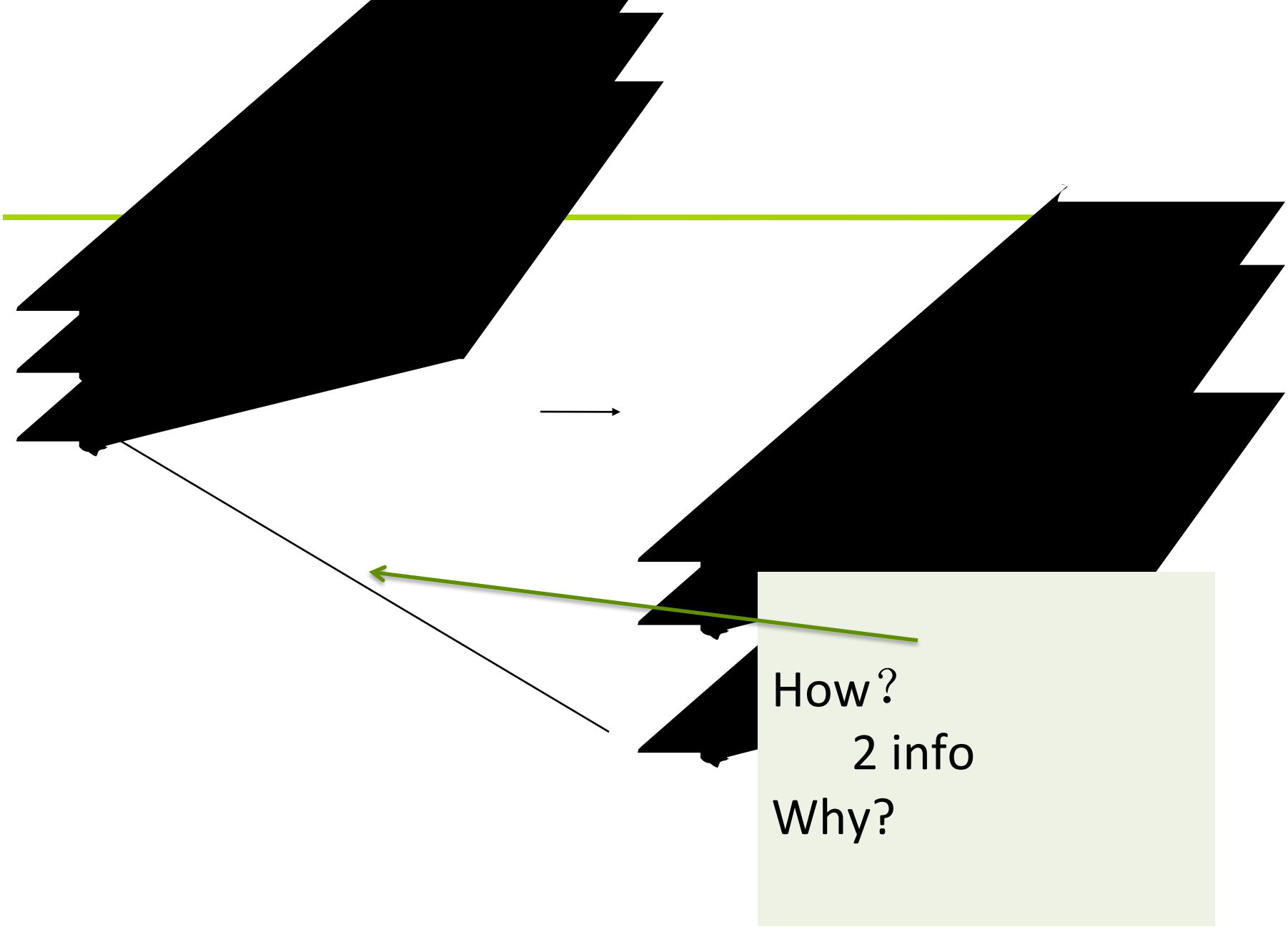
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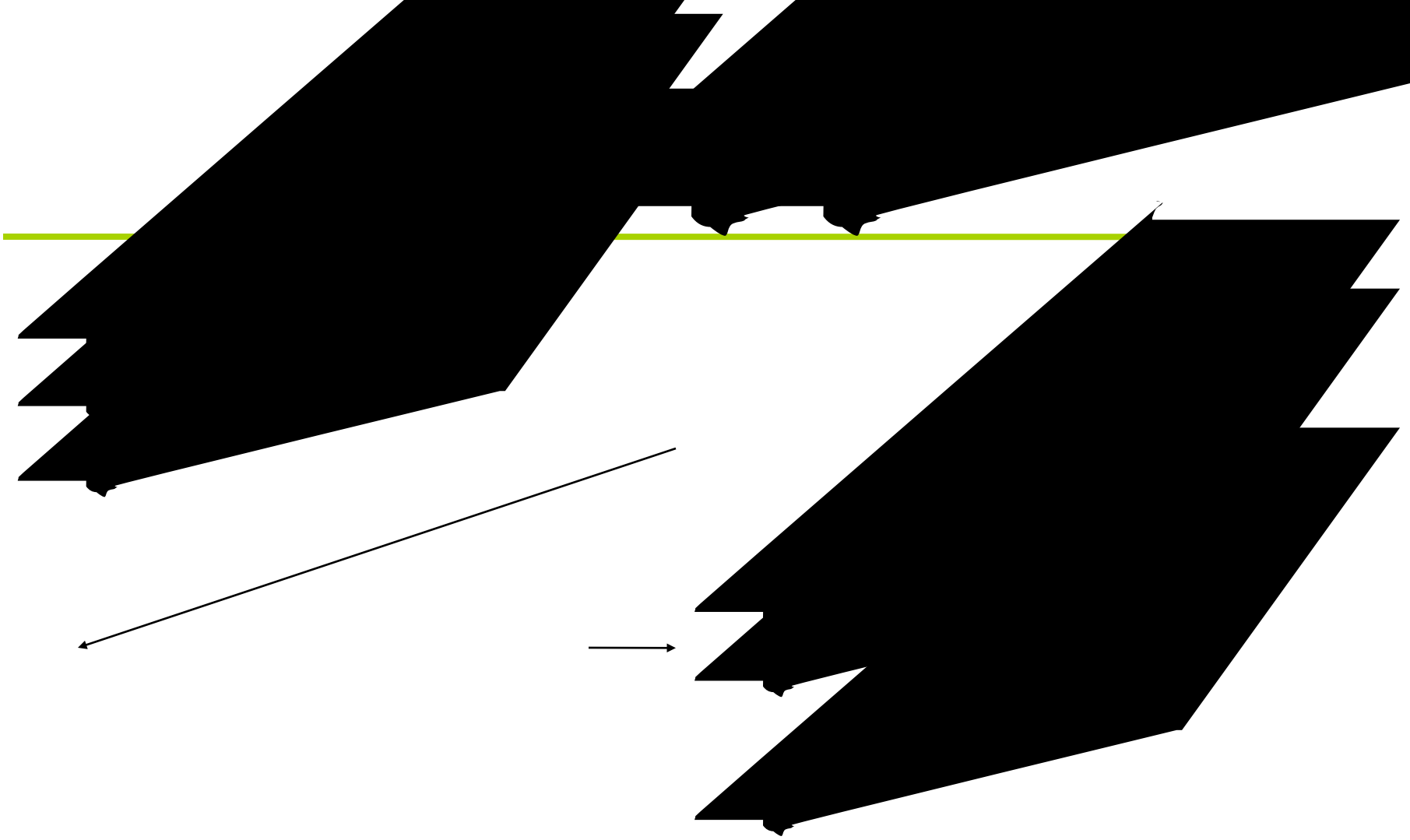

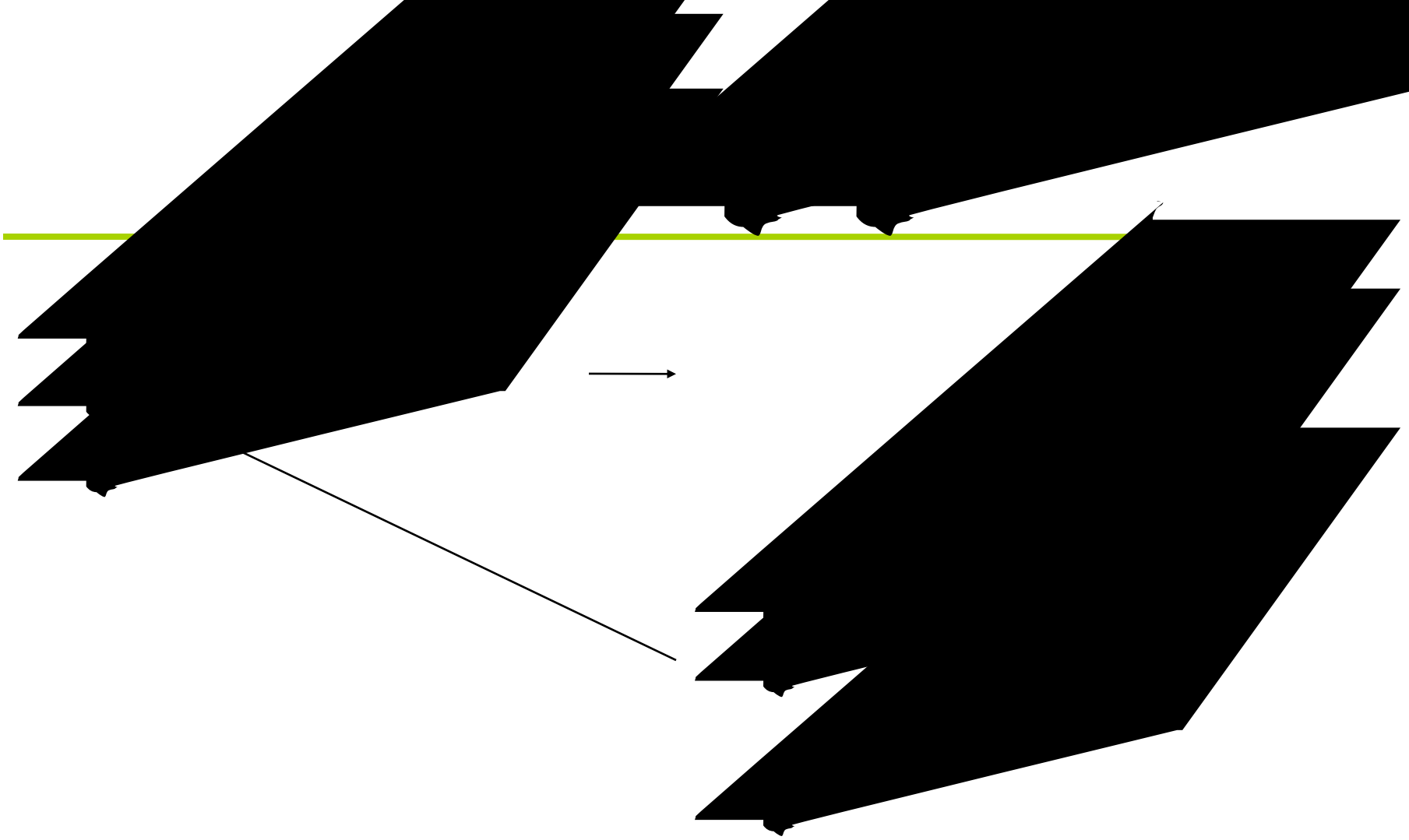

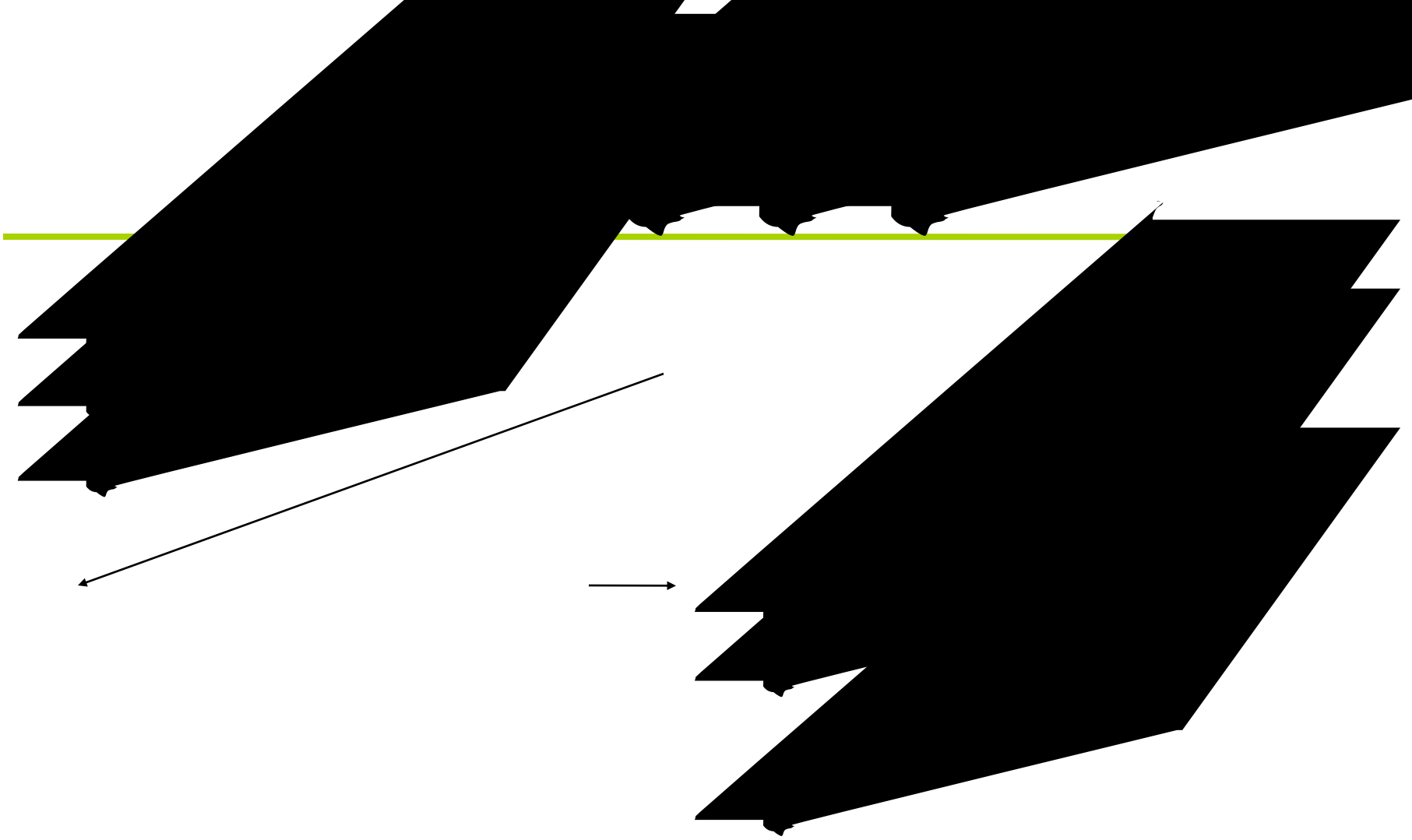

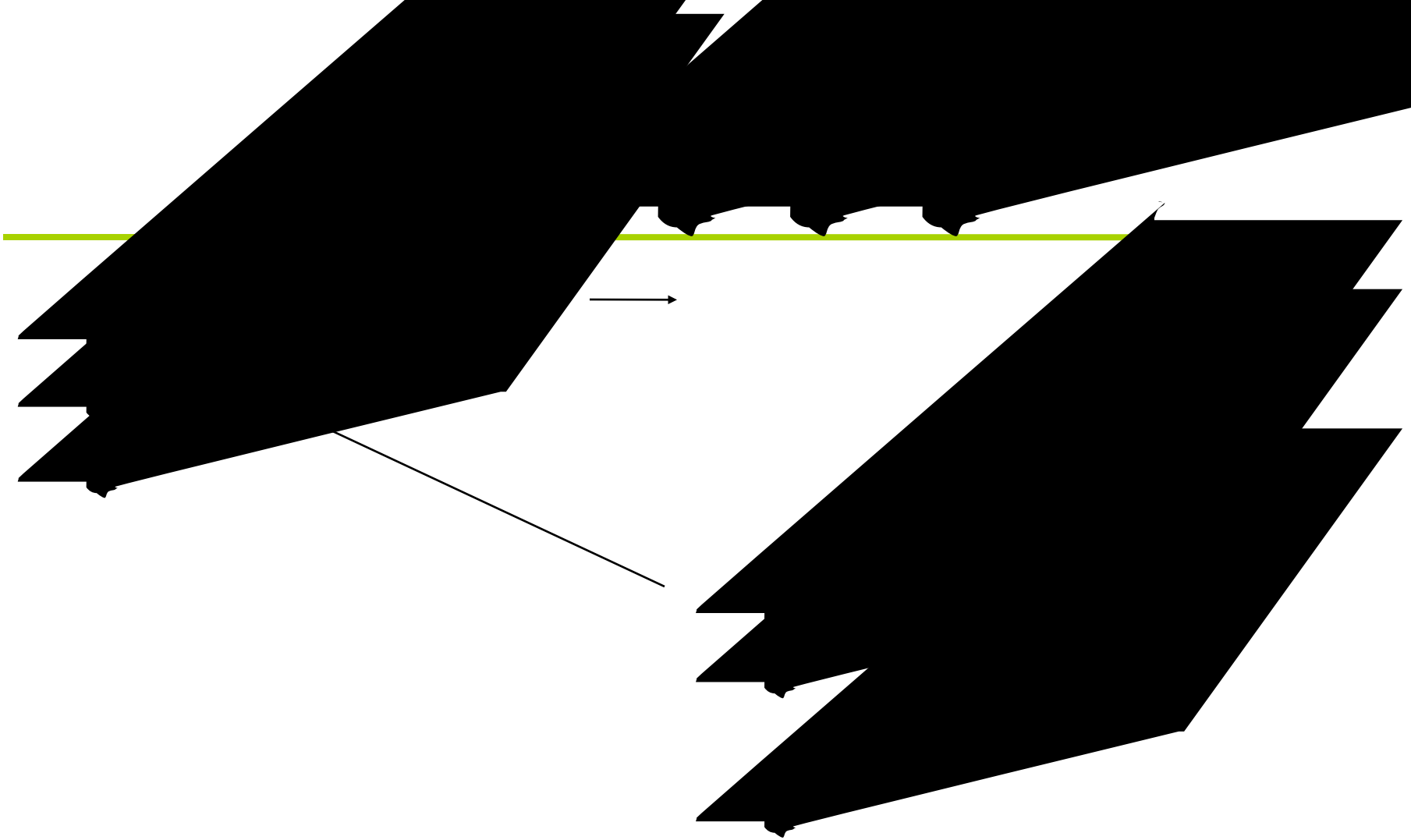

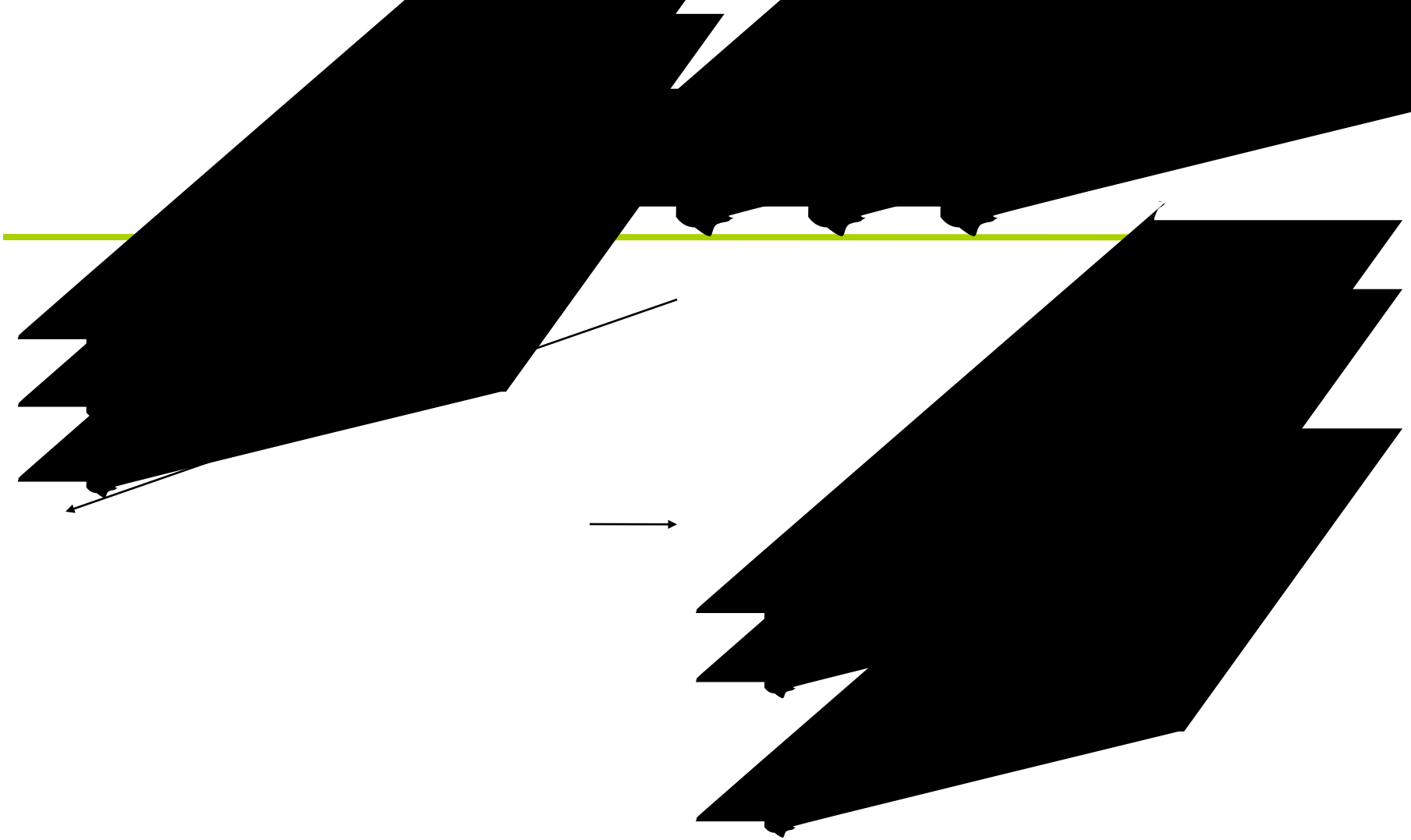

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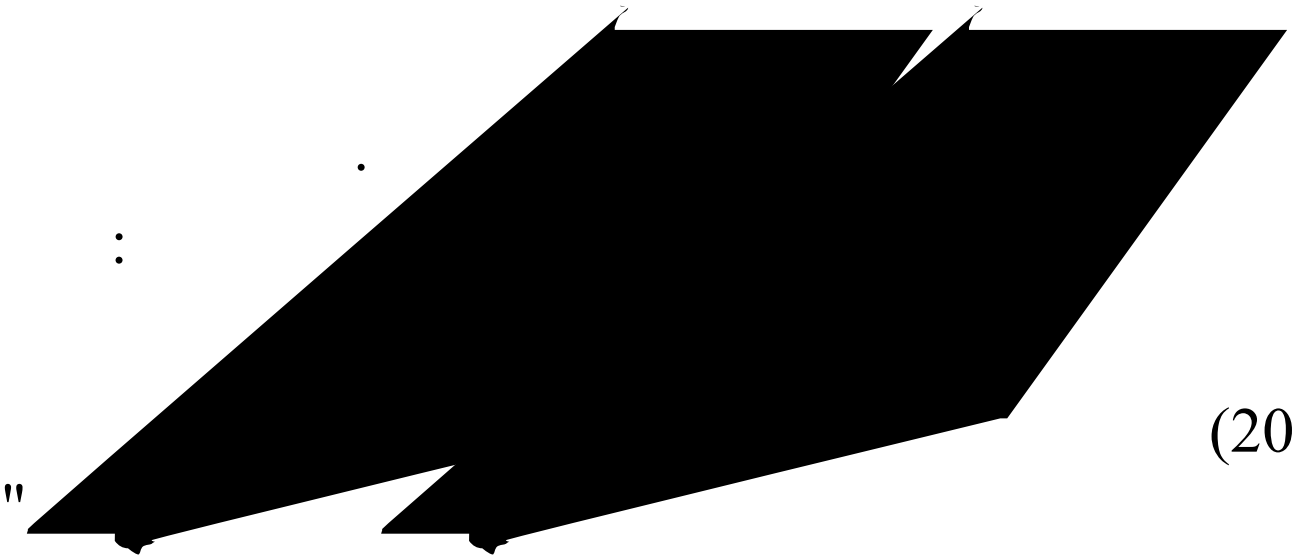

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