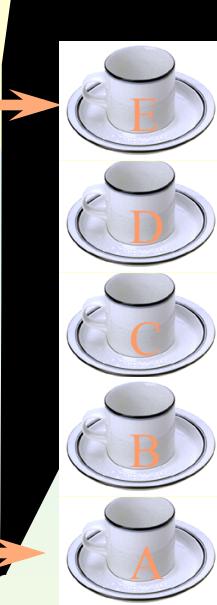


# Stacks and Queues

---

## The Stack Abstract Data type



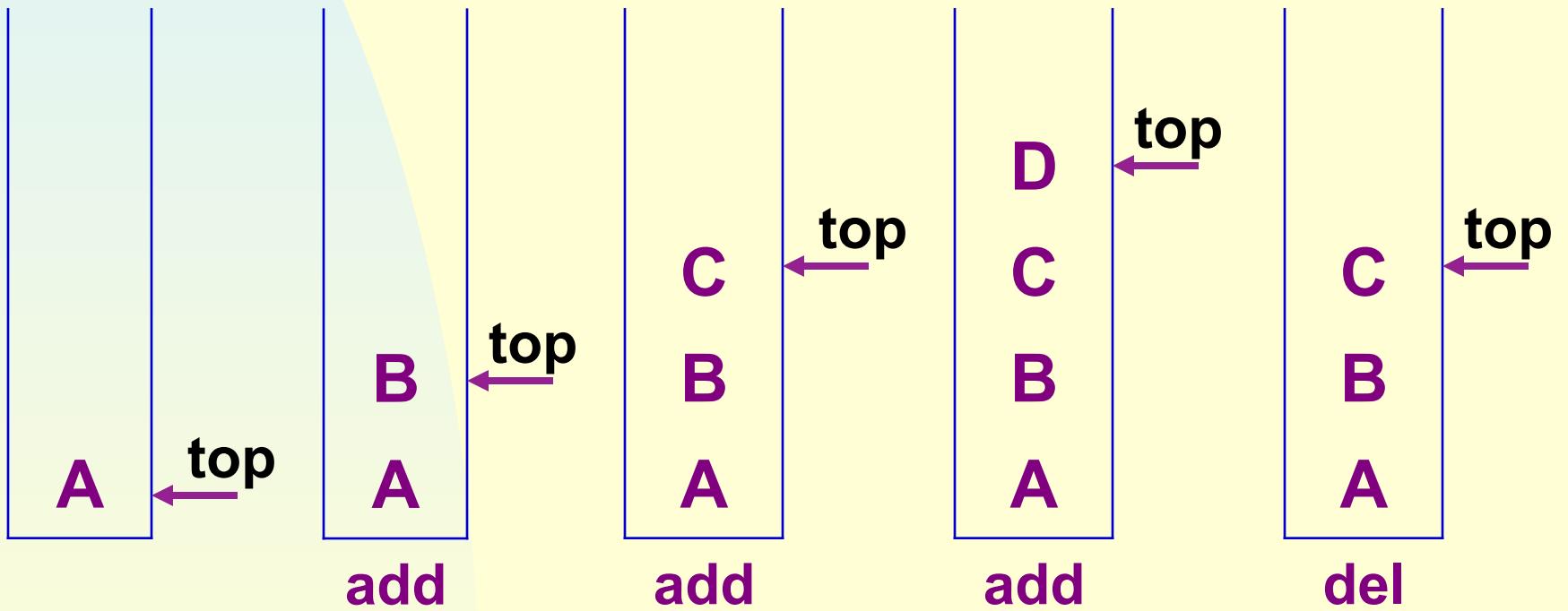
top

bottom

The diagram shows a vertical stack of five cups, each with a saucer. The cups are labeled from bottom to top as A, B, C, D, and E. An orange arrow points to the top cup with the label "top". Another orange arrow points to the bottom cup with the label "bottom".



Remove a cup from the stack.  
A stack is a LIFO list.



## ADT 3.1 Stack

```
<      T>
Stack
// A finite ordered list  ith  ero or more elements.
:
Stack (  stackCapacit  = 10);
//Creates an empt  stack  ith initial capacit  of stackCapacit

    IsEmpt ()      ;
//If number of elements in the stack is 0,      else

    T& Top()      ;
// Return the top element of stack

    Push(      T& item);
// Insert item into the top of the stack
    Pop();
// Delete the top element of the stack.

;
```

:

T\* stack;  
top;  
capacit ;

,

< T>

Stack<T>::Stack( stackCapacit ): capacit (stackCapacit )

(capacit < 1) Stack capacit must be > 0 ;  
stack = T[capacit ];  
top = -1;

< T>

Stack<T>::IsEmpty ()

(top == -1);

< T>  
T& Stack<T>::Top()

(IsEmpty ) Stack is Empt ;  
stack[top];

< T>  
Stack<T>::Push( T& )

(top == capacit - 1)

ChangeSi e1D(stack, capacit , 2\*capacit );  
capacit \*= 2;

stack[++top] = ;

**1**

**1-**

:

< T>

ChangeSi e1D(T\* a, oldSi e, ne Si e)

(ne Si e < 0) Ne length must be  $\geq 0$  ;

T\* temp = T[ne Si e];

number = min(oldSi e, ne Si e);

cop (a, a + number, temp);

[] a;

a = temp;

```
<      T>
Stack<T>::Pop()
// Delete top element of stack.
(IsEmpt ())           Stack is empt , cannot delete. ;
stack[top--]. T(); // destructor for T
```

: 138-1, 2

Bus  
Stop



front

rear

rear

rear

rear



Bus  
Stop



front

rear



Bus  
Stop



front

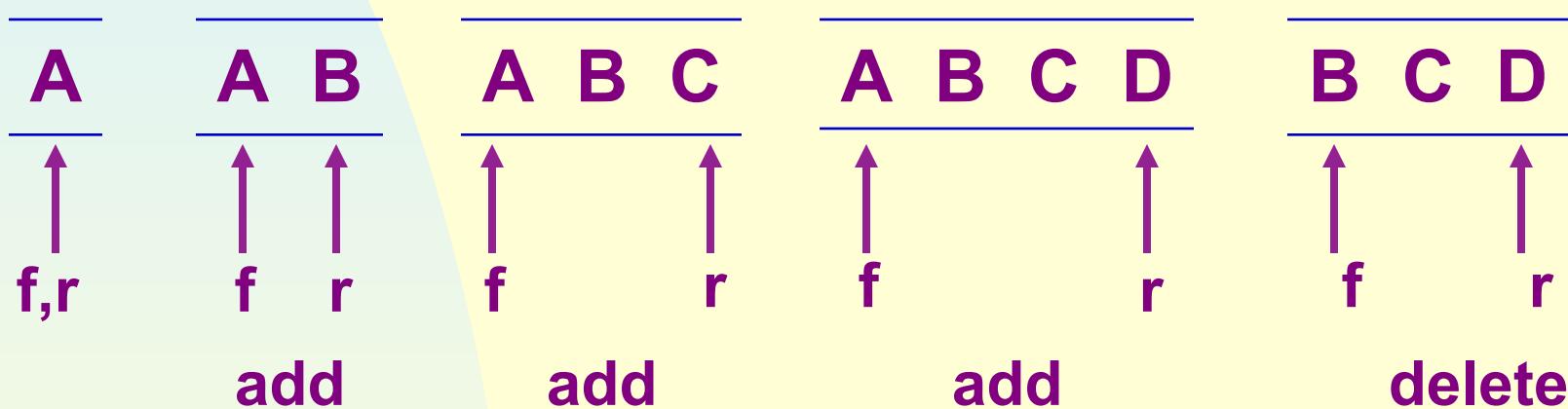
rear

Bus  
Stop



## 3.3 The Queue Abstract Data Type

### 3.3 The Queue Abstract Data Type



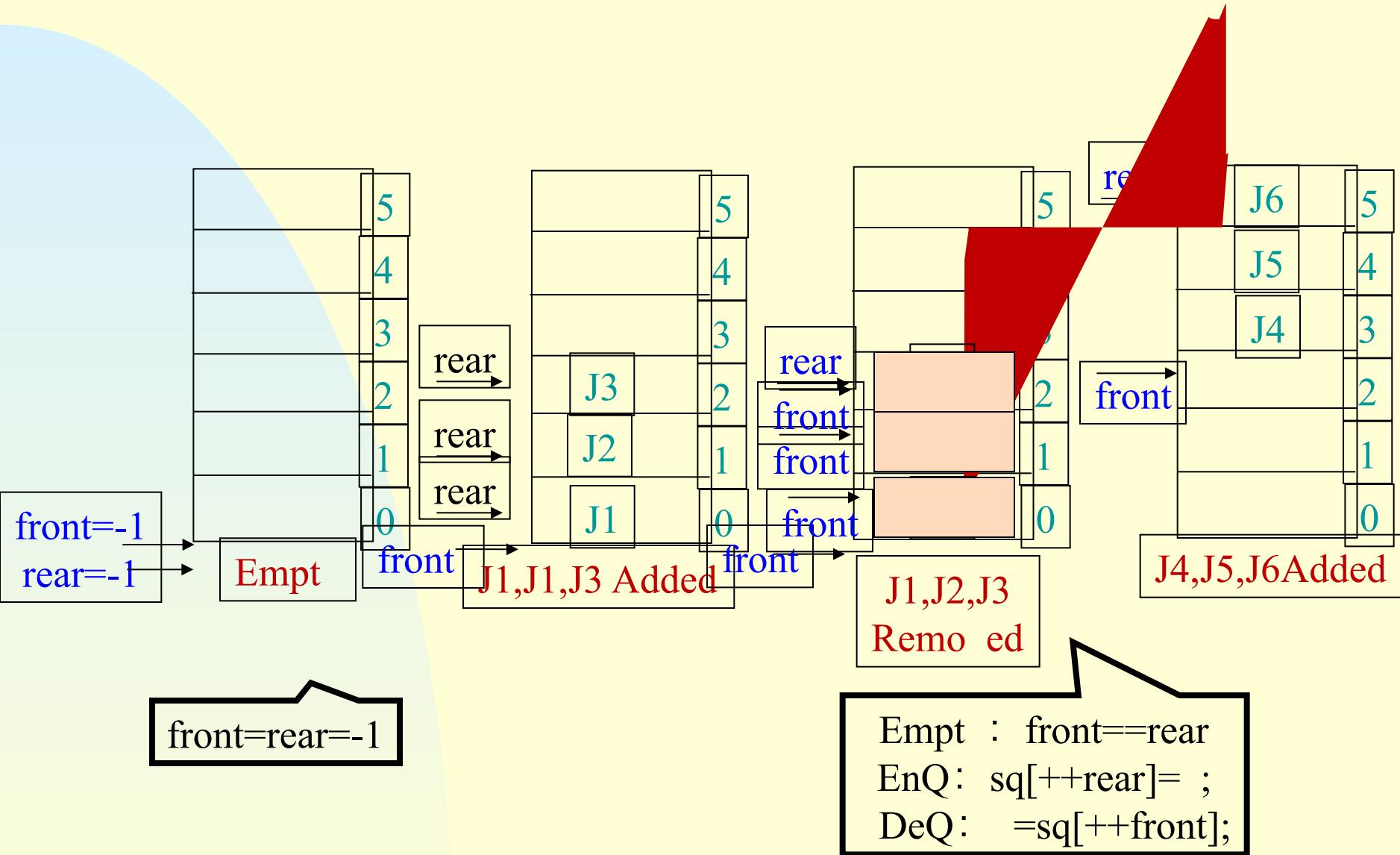
$f = q[e]$  front     $r = q[e]$  rear

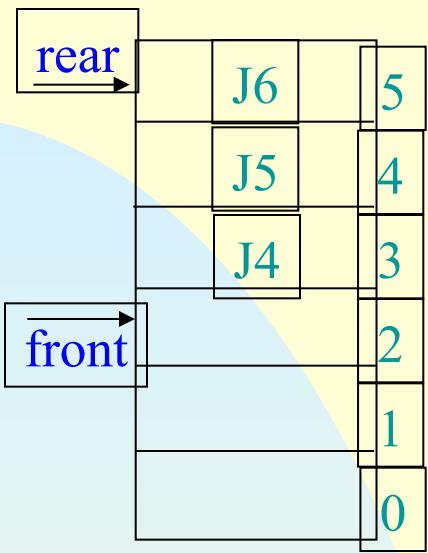
```
<      T>
Queue
// A finite ordered list  ith  ero or more elements.
:
Queue (  queueCapacit  = 10);
// Creates an empt  queue  ith initial capacit  of
// queueCapacit
IsEmpty
```

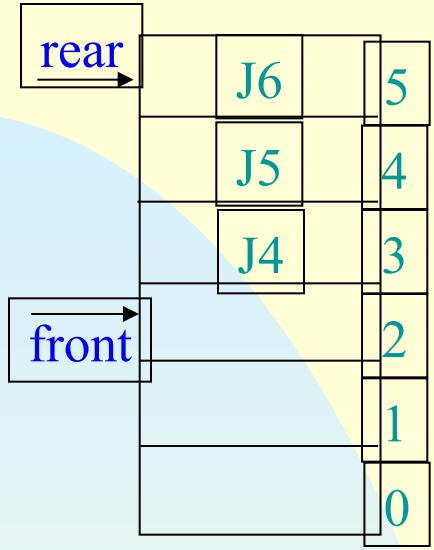
```
    :  
T* queue;  
front,  
rear,  
capacit ;
```

,

:







◆  $6 \rightarrow 2$

◆  $6 \rightarrow 1$

◆  $6 \rightarrow 0$

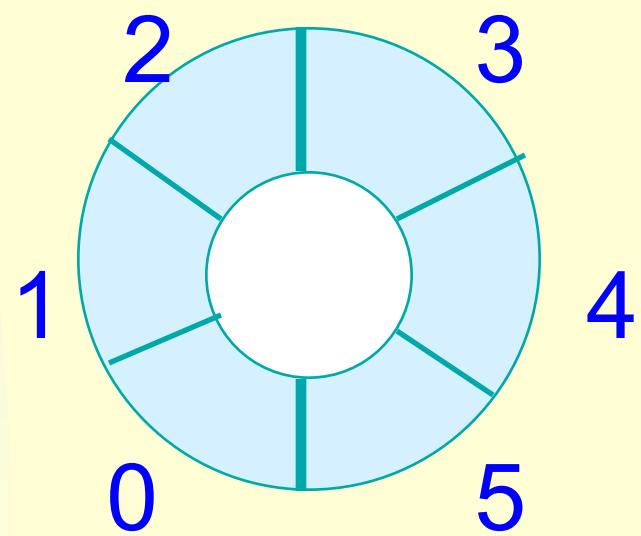
6

6 % 6 0

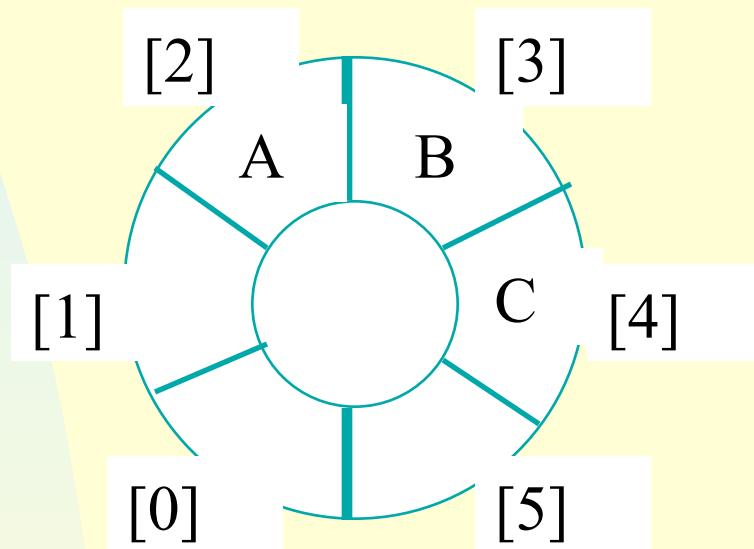
■

1

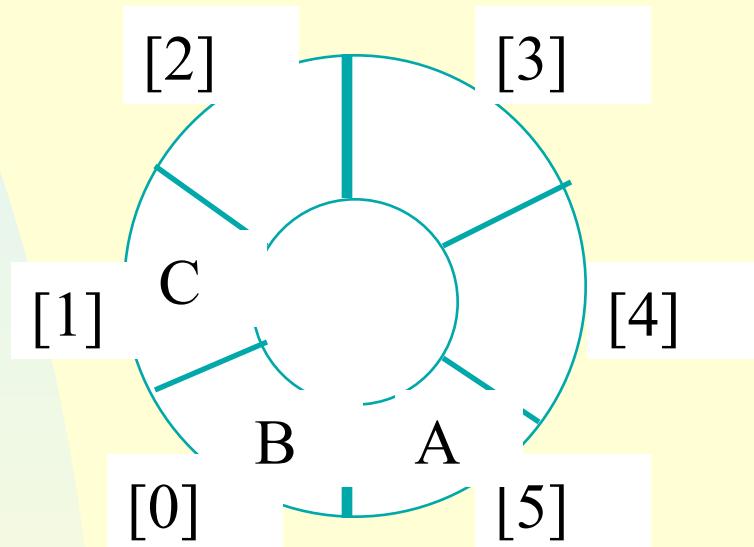
queue[]



Possible configuration with 3 elements.



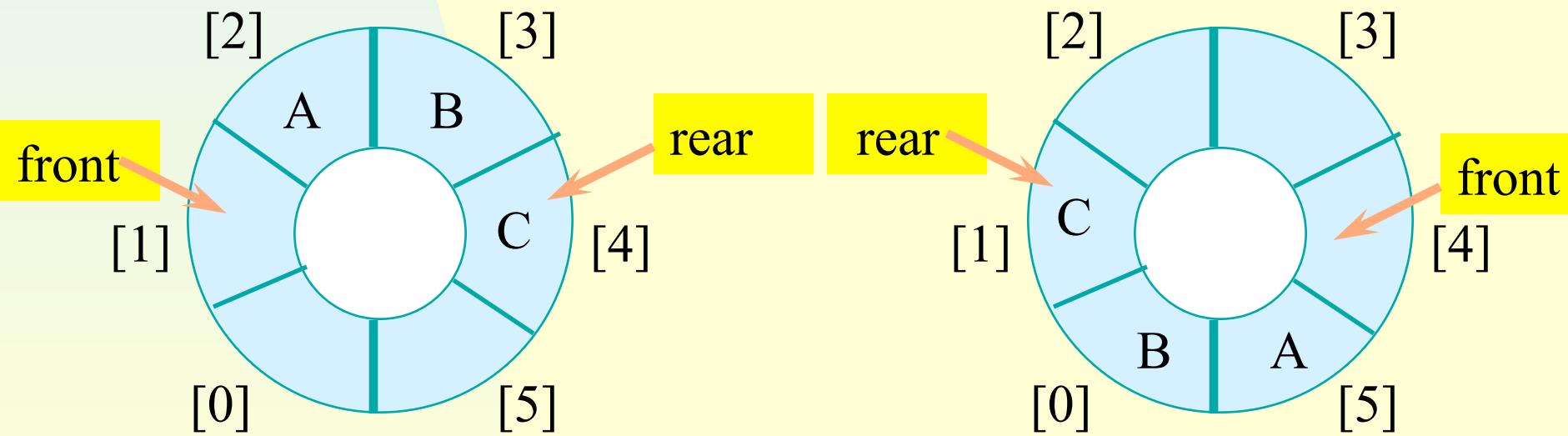
Another possible configuration with 3 elements.



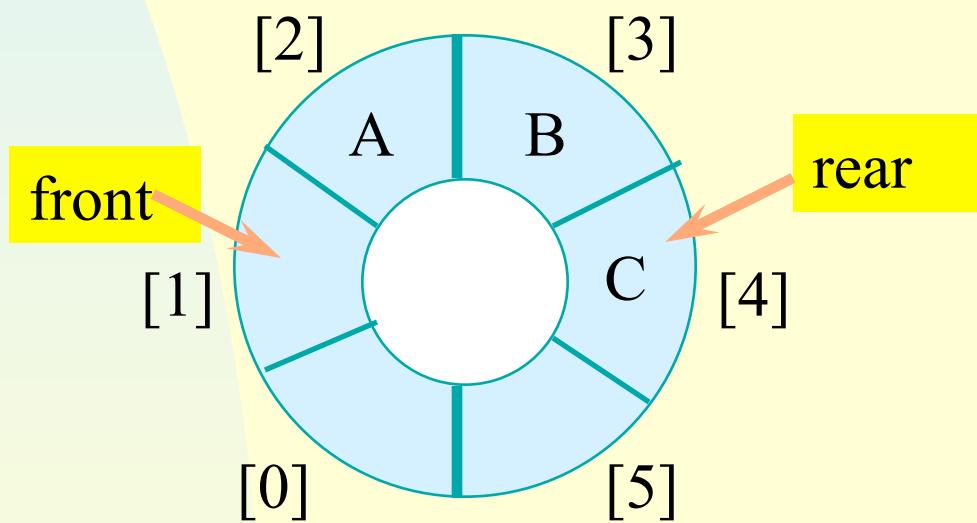
Use integer variables **front** and **rear**.

**front** is one position counterclockwise from first element

**rear** gives position of last element

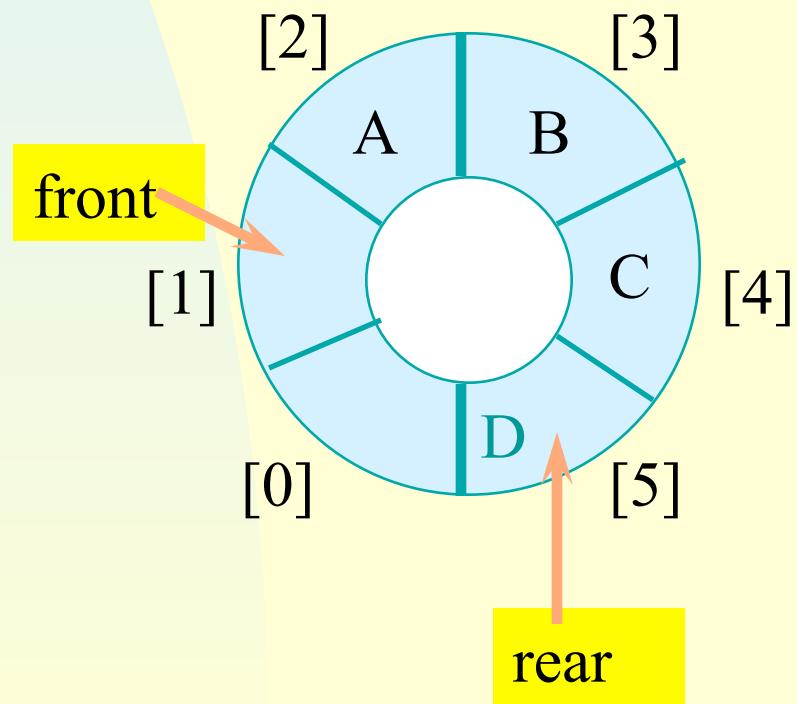


Move rear one clock wise.

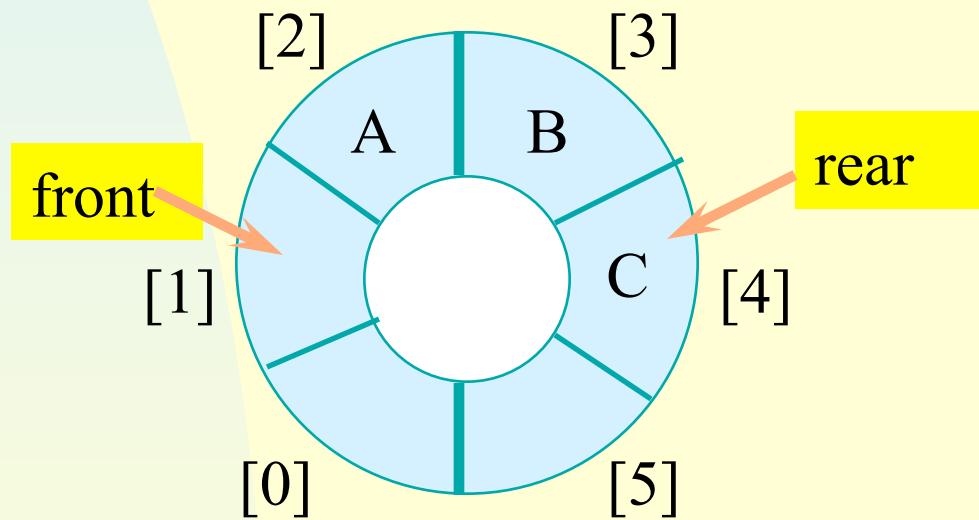


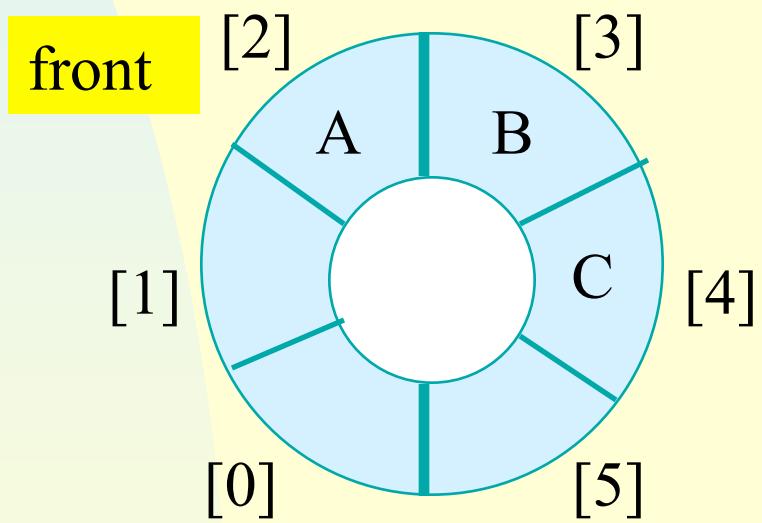
Move rear one clock wise.

Then put into `queue[rear]`.



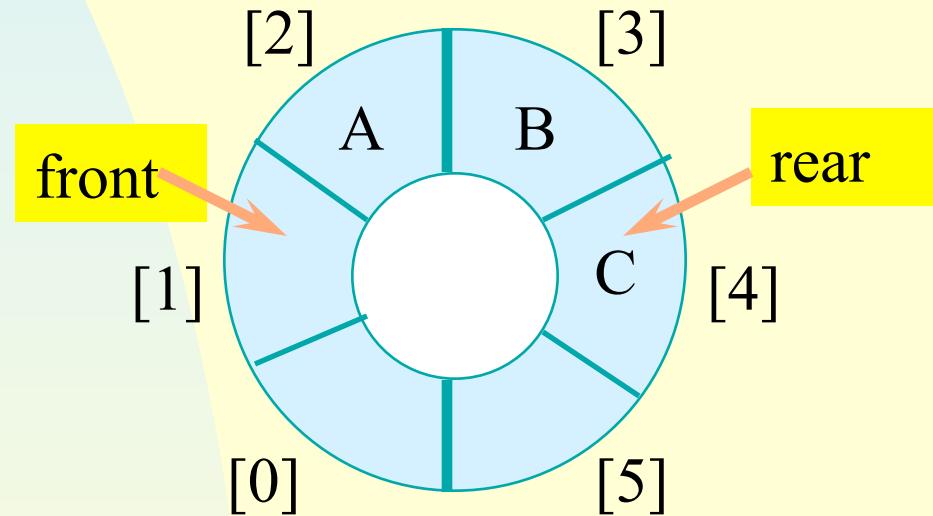
Move front one clockwise wise.



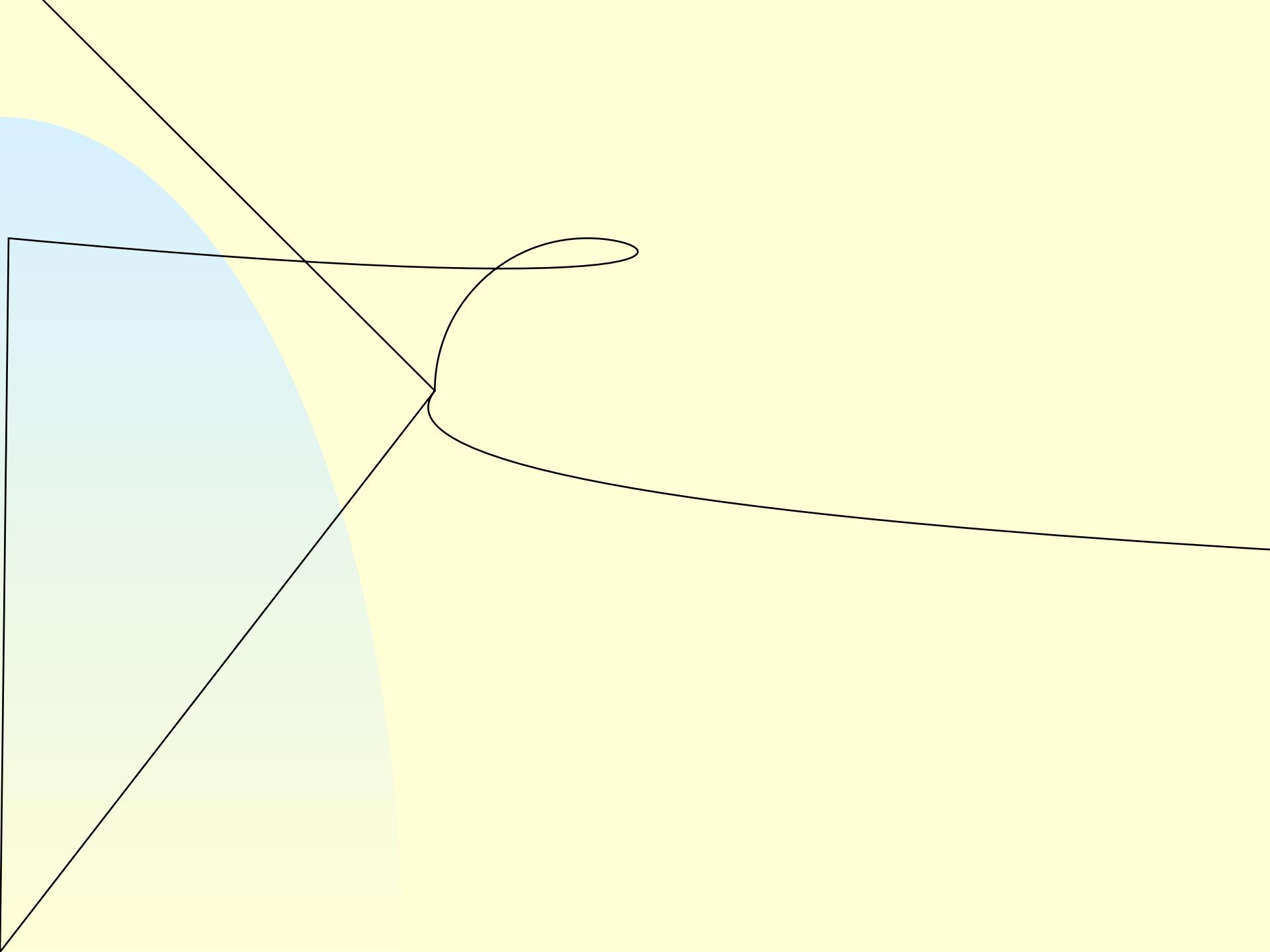


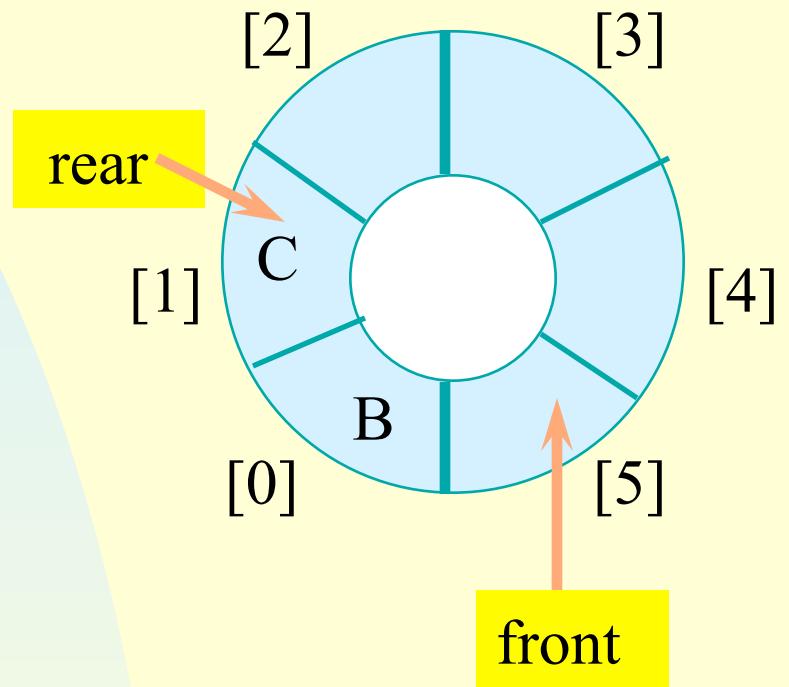
`rear++;`

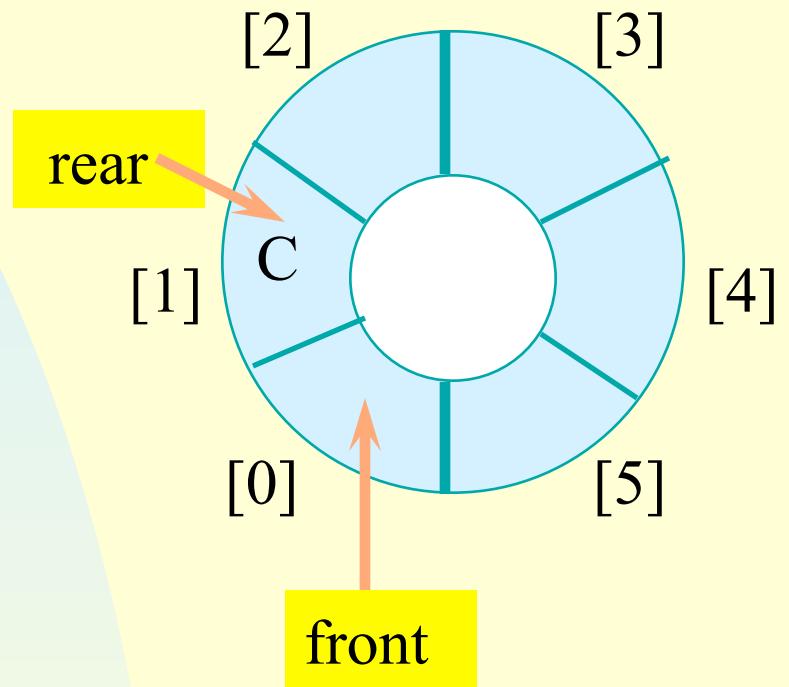
`if (rear == queue.length) rear = 0;`

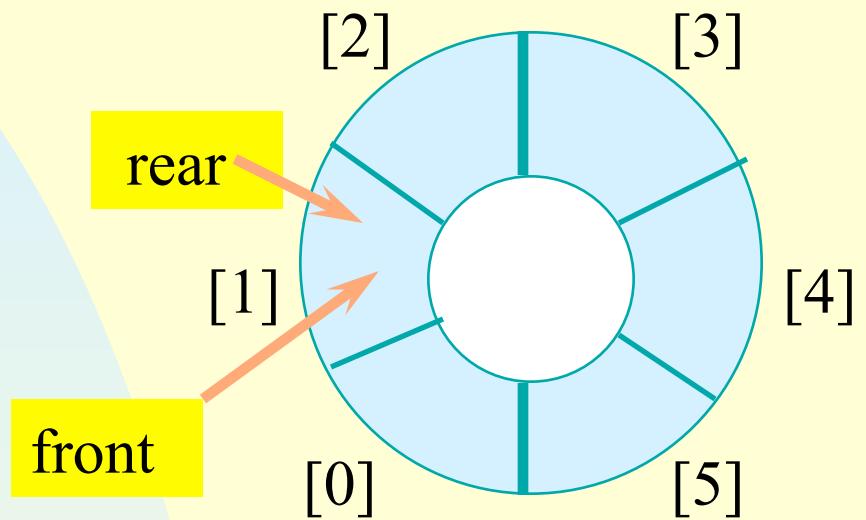


`rear = (rear + 1) % queue.length;`

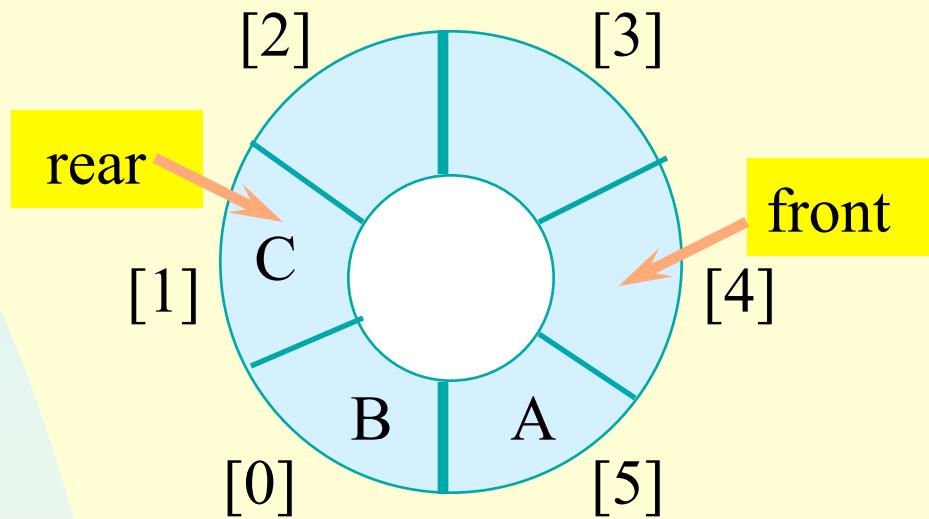


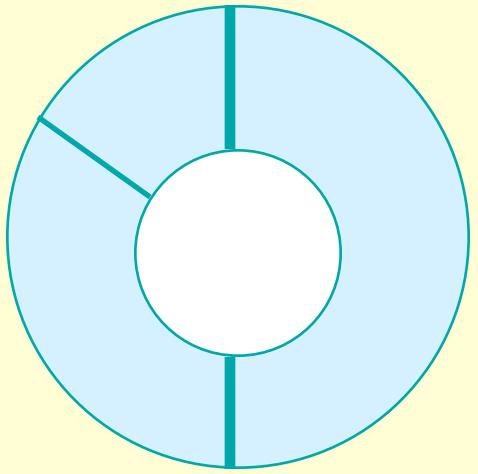


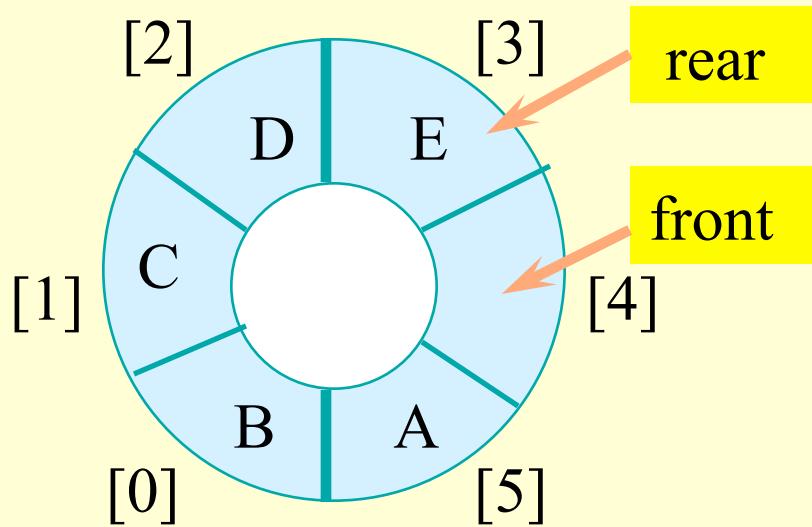


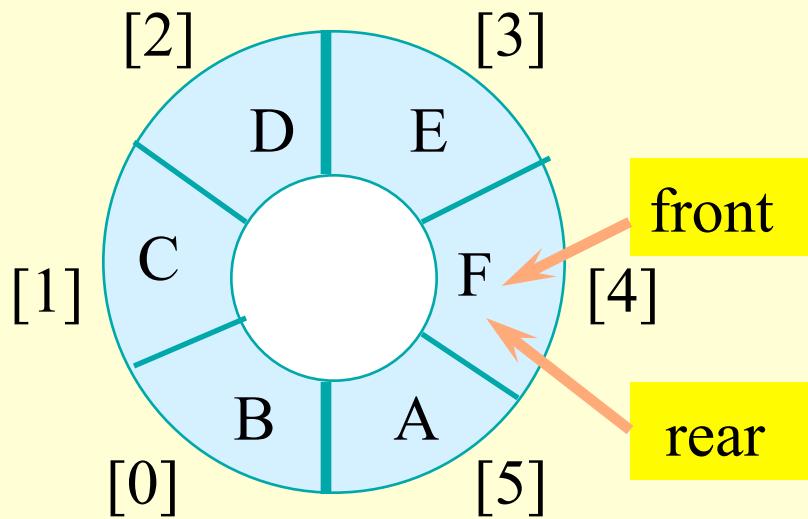


0.



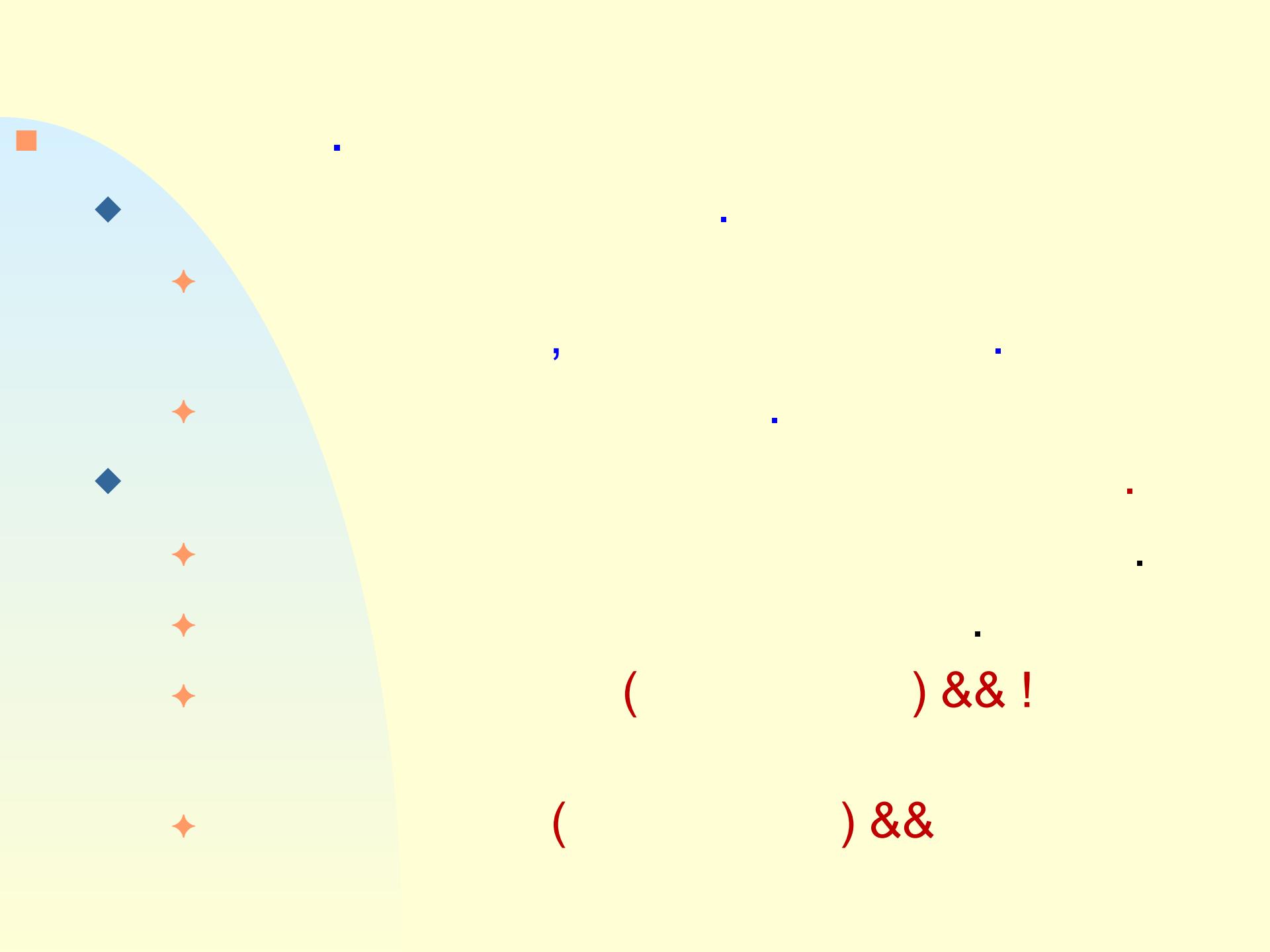






When a series of adds causes the queue to become full, front = rear.

So we cannot distinguish between a full queue and an empty queue!



( ) && !

( ) &&



```
<      T>
Queue<T pe>::Queue(    queueCapacit ):
                           capacit (queueCapacit )

(capacit < 1)           Queue capacit must > 0 ;
queue =     T[capacit ];
front = rear = 0;
```

```
<      T>
    Queue<T>::IsEmpty()
front==rear ;
```

```
<      T>
T& Queue<T>::Front()
```

(IsEmpty ())              Queue is empt . No front element ;  
queue[(front+1)%capacit ];

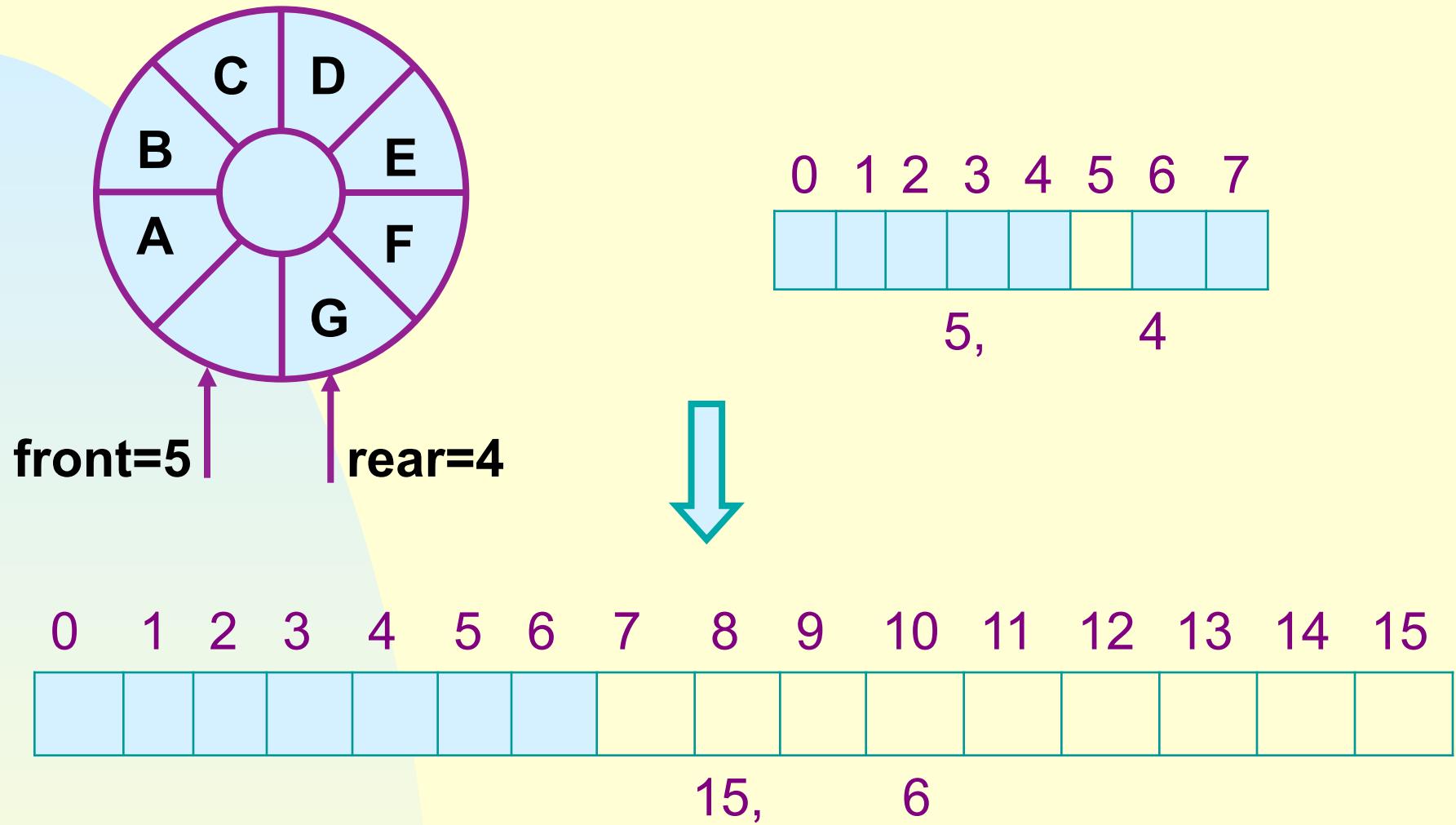
```
<      T>
T& Queue<T>::Rear()
```

(IsEmpty ())              Queue is empt . No rear element ;  
queue[rear];

```
<      T>
Queue<T>::Push(      T&  )
// add  at rear of queue
((rear+1)%capacit == front)
// queue full, _____
// code to double queue capacit  comes here
```

```
rear = (rear+1)%capacit ;
queue[rear] = ;
```

:



(1)

(2)

(3)

0.

-

-1.

•

```
// allocate an array to store the capacity
T* Queue = T[2*capacity];

// copy from queue to new Queue
start = (front+1)%capacity;
if (start < 2)
    // no wrap around
    cop(queue+start, queue+start+capacity -1, new Queue);

// queue wraps around
cop(queue+start, queue+capacity, new Queue);
cop(queue, queue+rear+1, new Queue+capacity -start);

// switch to new Queue
front = 2*capacity -1; rear = capacity -2; capacity *= 2;
[] queue;
queue = new Queue;
```

```
<      T>
Queue<T>::Pop()
// Delete front elemnet from queue
(IsEmpt ())           Queue is empt . Cannot delete ;
front = (front+1)%capacit ;
queue[front]. T;
```

,

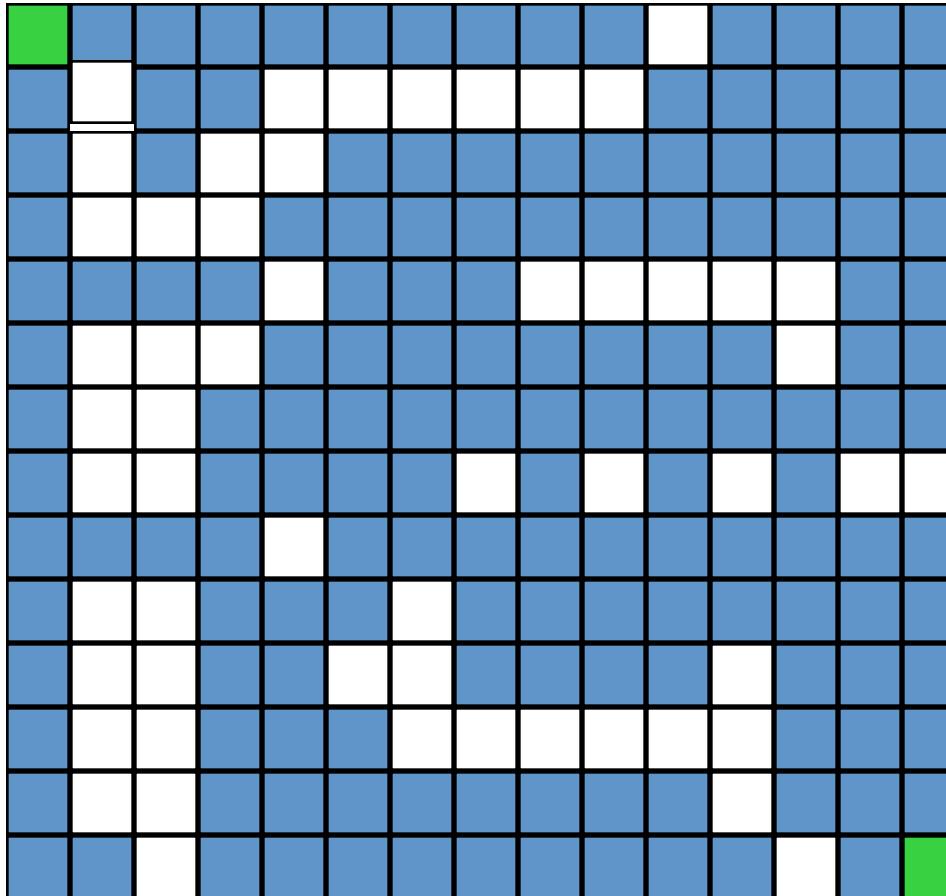
(

)

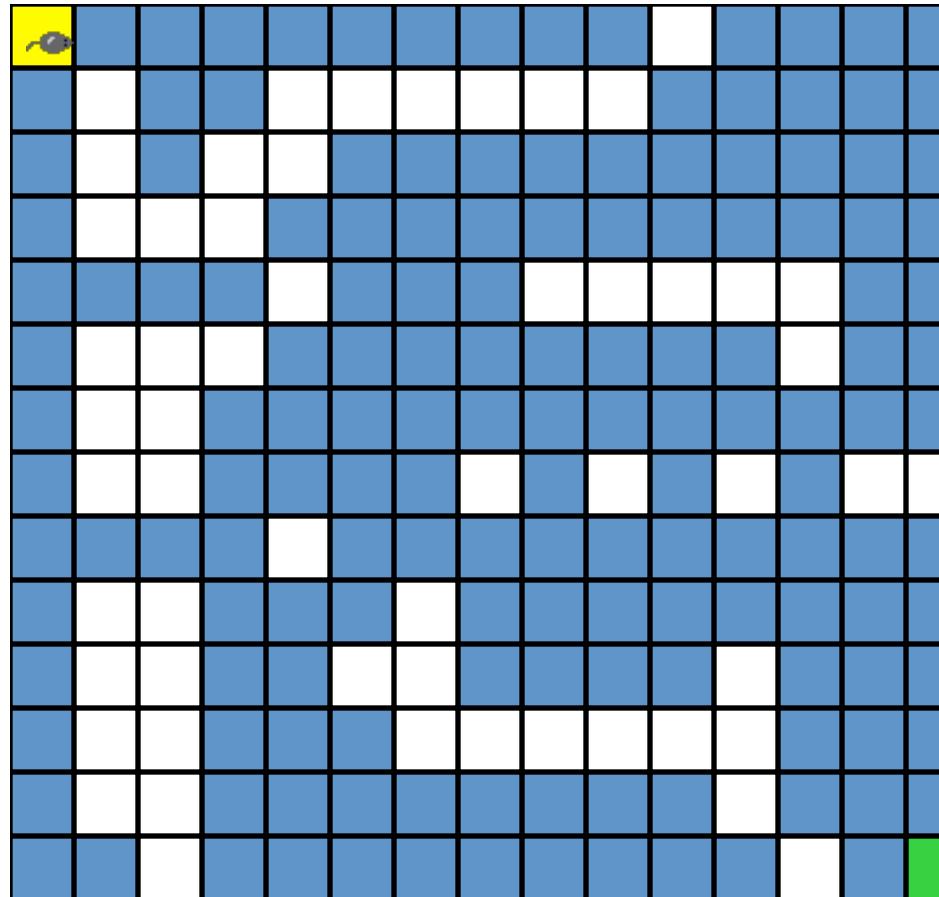
(1).

**E**rcises: P147-1, 3.

! " # \$ % & \$ ' \$ ( " ) \* \$



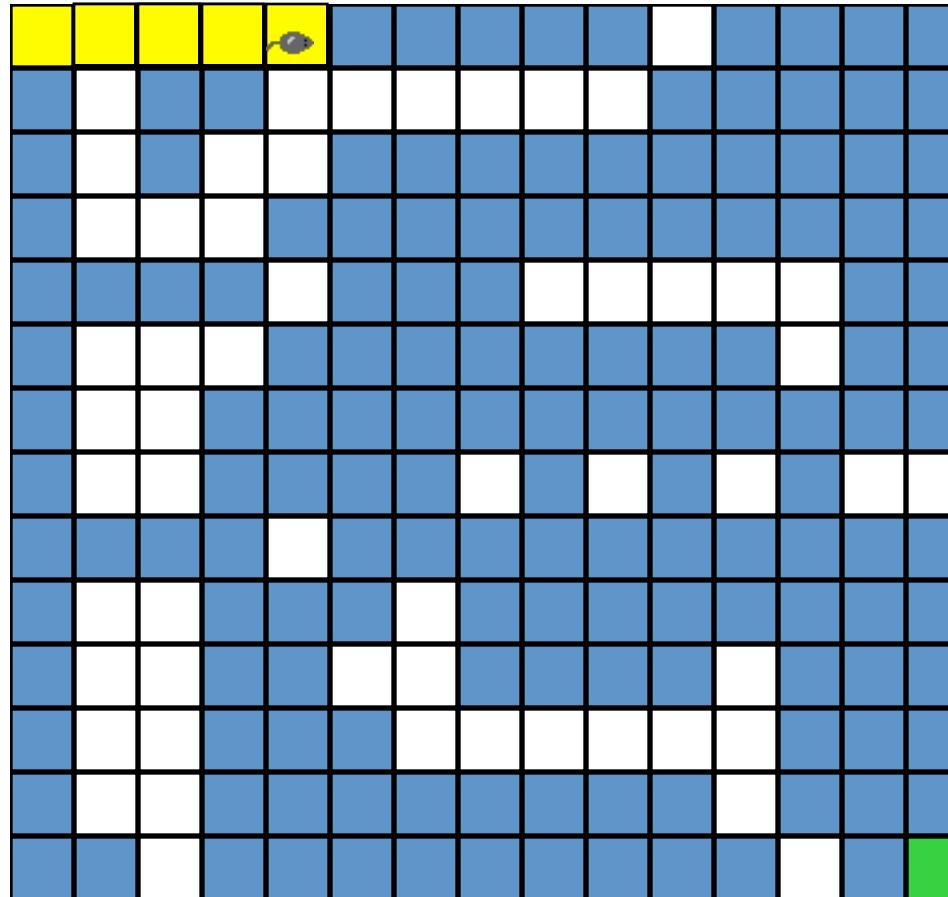
! " # \$ % & \$ ' \$ ( " ) \* \$



( + , \* \$ + - . \* - \$ / 01 \$ - / 23 # 4 \$ . + 5 & 4 \$ 6 \* 7 4 \$ 89 \$

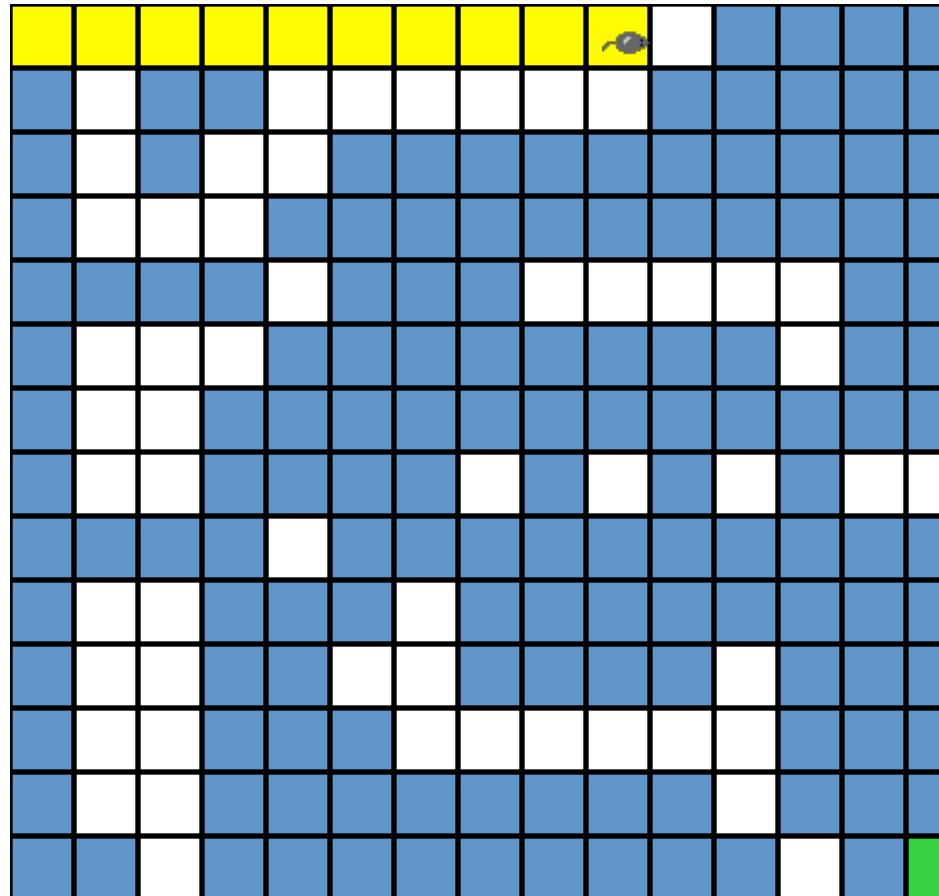
: 6 + ; < \$ 9 + 0 / = + & 0 \$ # + \$ " , + / . \$ - \* , / 0 / # > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



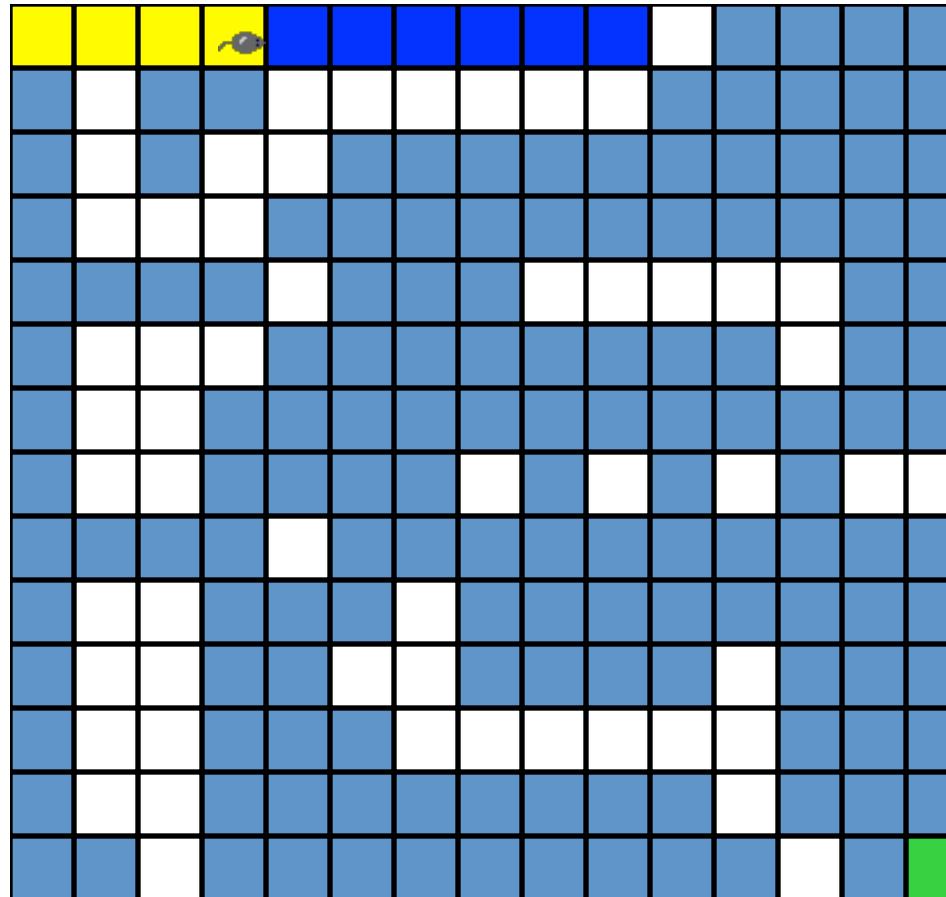
( + , \* \$ + - . \* - \$ / 01 \$ - / 23 # 4 \$ . + 5 & 4 \$ 6 \* 7 4 \$ 89 \$  
: 6 + ; < \$ 9 + 0 / = + & 0 \$ # + \$ " , + / . \$ - \* , / 0 / # \$

! " # \$ % & \$ ' \$ ( " ) \* \$



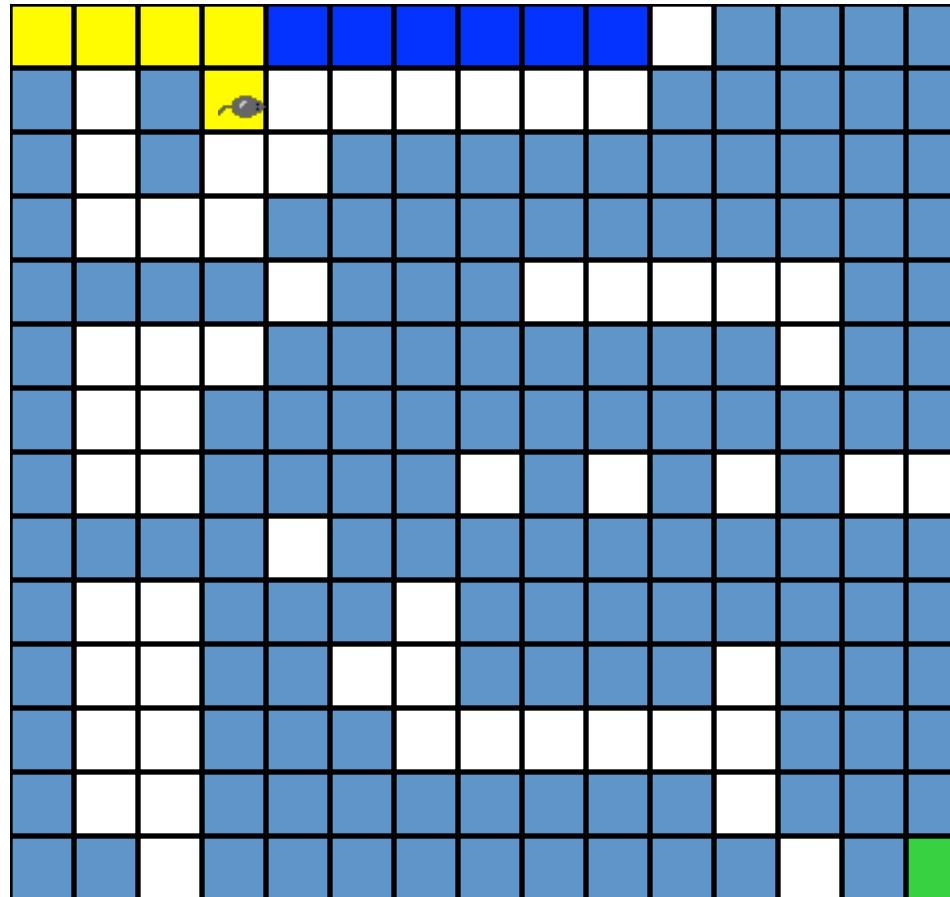
( + , \* \$ ? " ; < 5 " - . \$ 8 & = 6 \$ 5 \* \$ - \* " ; 3 \$ " \$ 0 @ 8 " - \* \$ A - + B \$ 5 3 / ; 3 \$ " \$  
A + - 5 " - . \$ B + , \* \$ / 0 \$ 9 + 0 0 / ? 6 \* > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



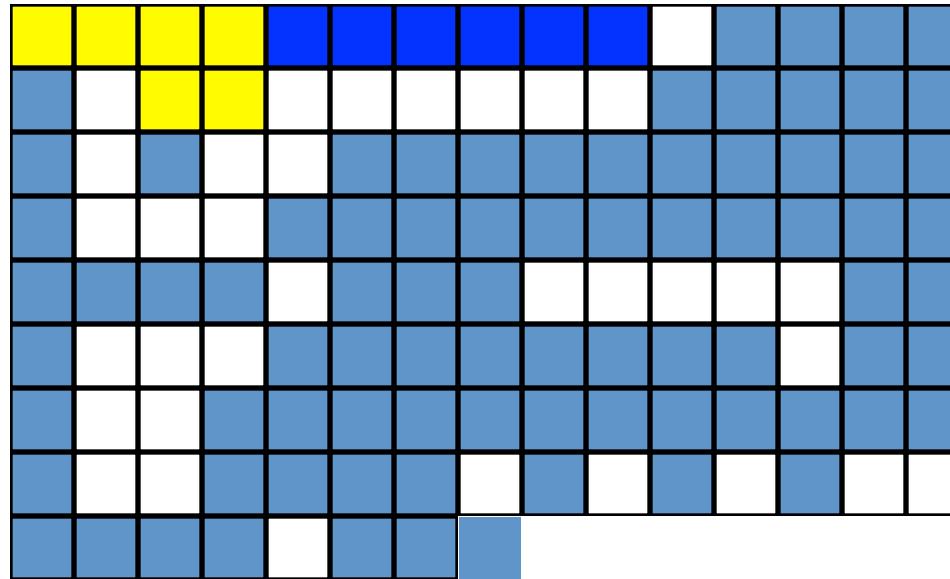
( + , \* \$. + 5 & > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



( + , \* \$ 6 \* 7 > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



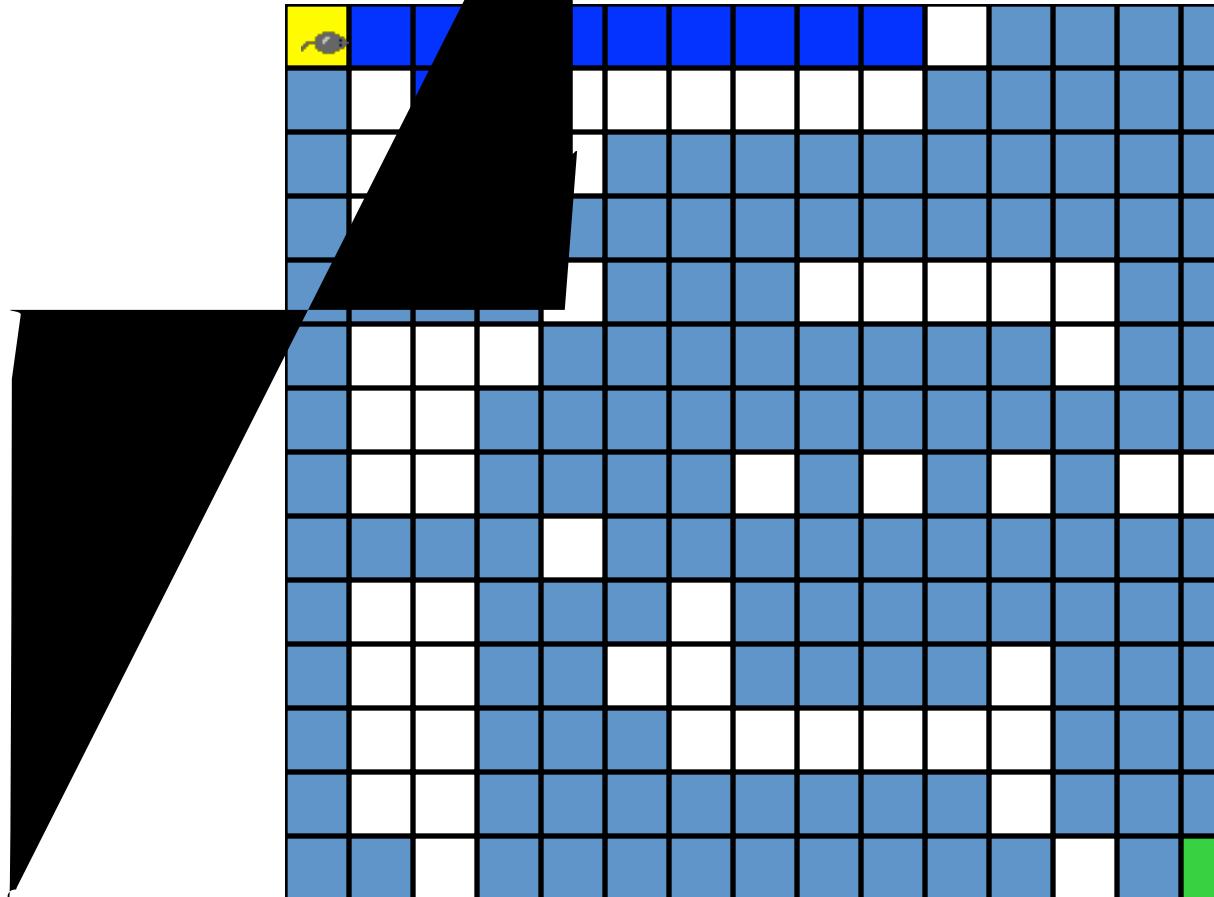
( + , \* \$. + 5 & > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



( + , \* \$ ? " ; < 5 " - . \$ 8 & = 6 \$ 5 \* \$ - \* " ; 3 \$ " \$ 0 @ 8 " - \* \$ A - + B \$ 5 3 / ; 3 \$ " \$  
A + - 5 " - . \$ B + , \* \$ / 0 \$ 9 + 0 0 / ? 6 \* > \$

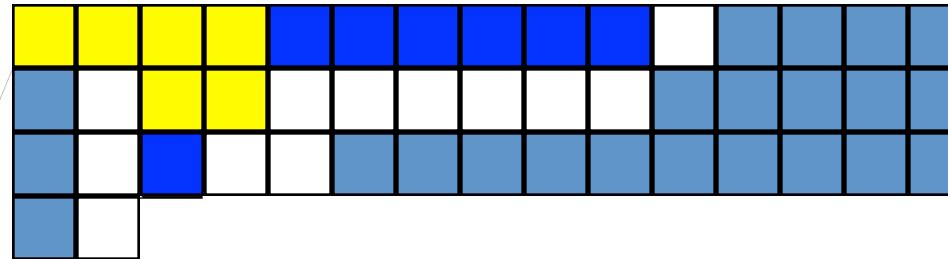
! # \$ % & \$ ' \$ ( " ) \* \$



( + , \* \$ ? " ; < 5 " - . \$ 8 & = 6 \$ 5 \* \$ - \* " ; 3 \$ " \$ 0 @ 8 " - \* \$ A - + B \$ 5 3 / ; 3 \$ " \$ A + - 5 " - . \$ B + , \* \$ / 0 \$ 9 + 0 0 / ? 6 \* > \$

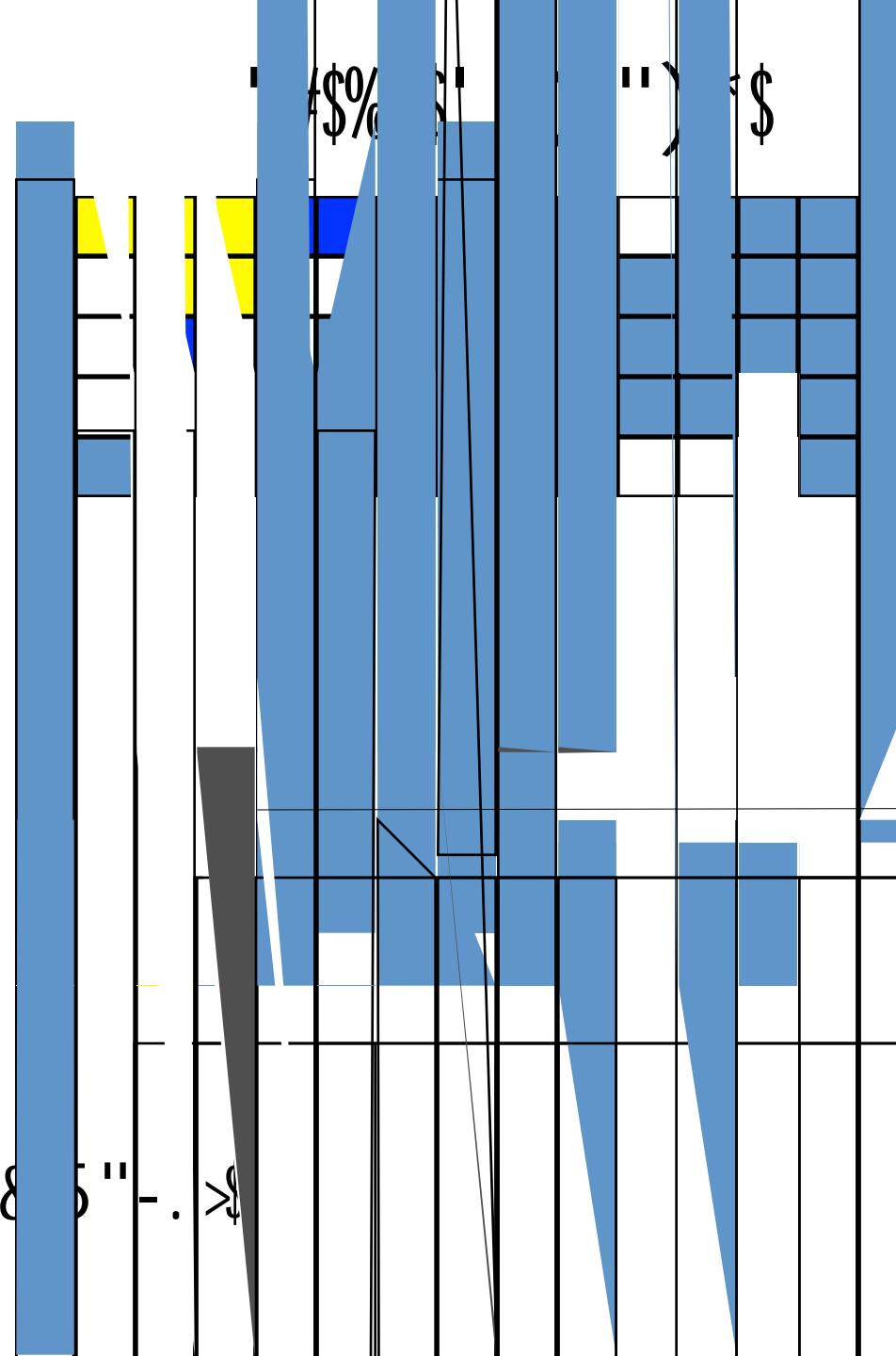
Mo e do n ard.

! " # \$ % & \$ ' \$ ( " ) \* \$



( + , \* \$ - / 23 # > \$

( +, \*\$. +585"- .

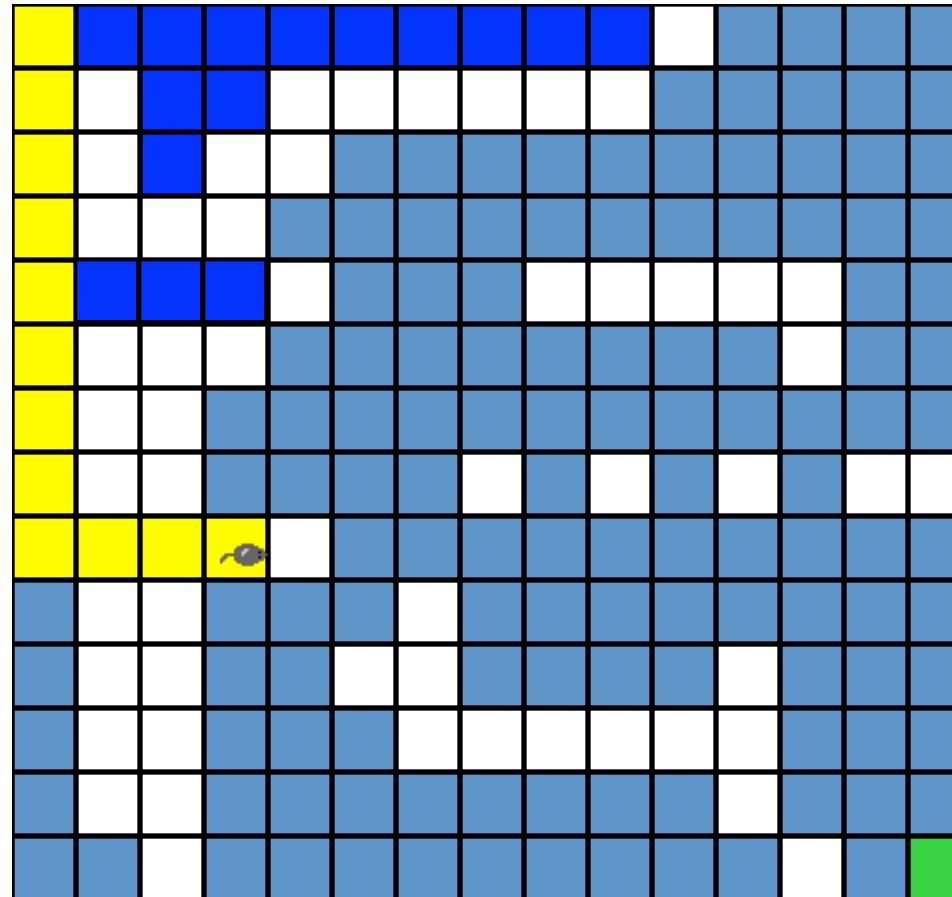


! " # \$ % & \$ ' \$ ( " ) \* \$



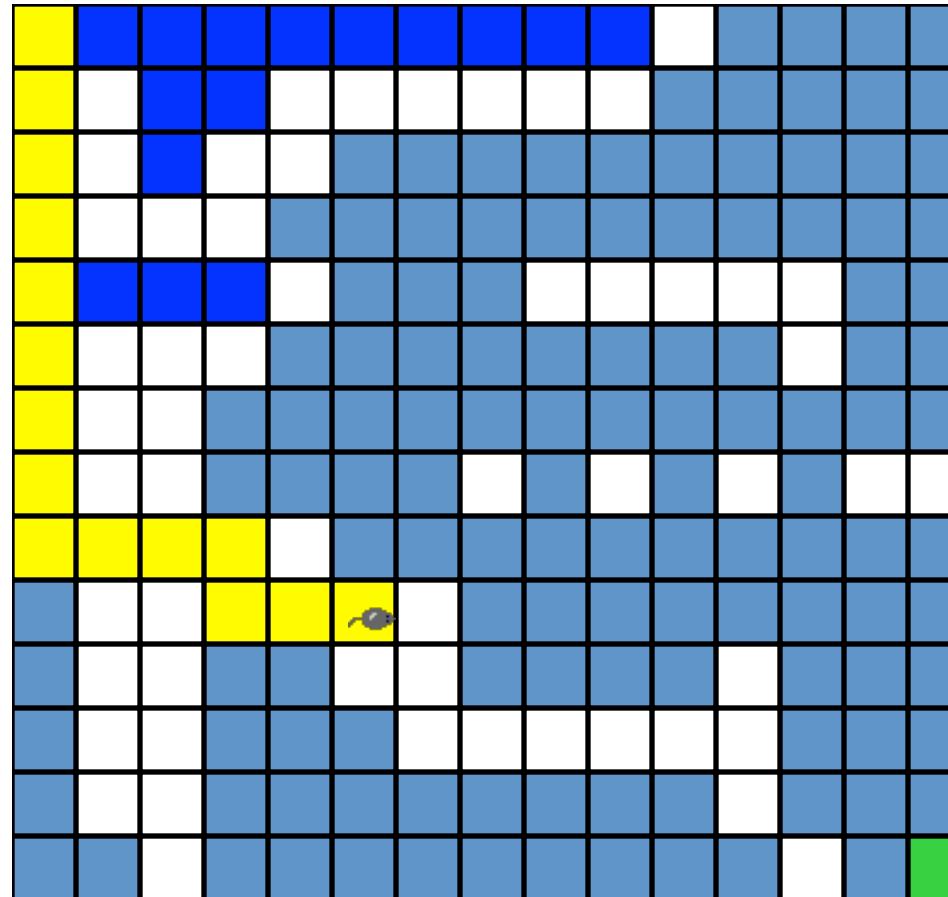
( + , \* \$ - / 23 # > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



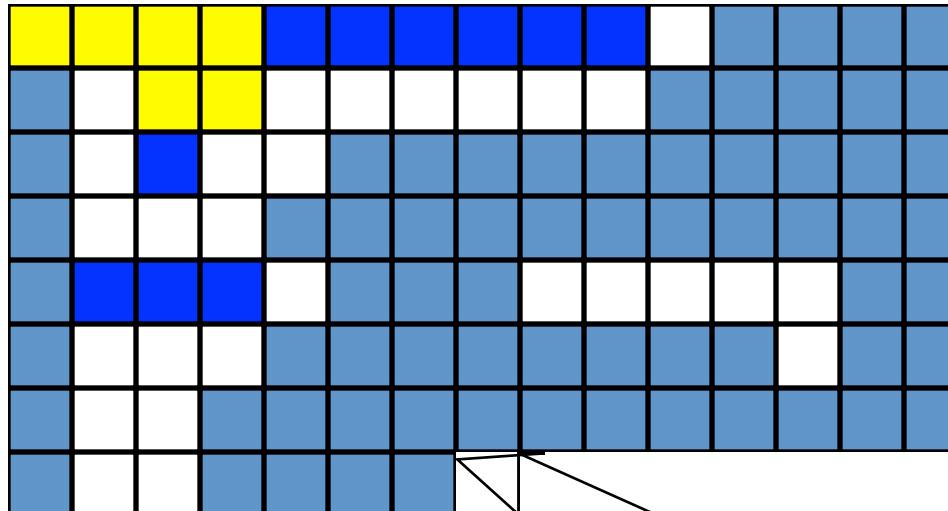
( + , \* \$ + & \* \$ . + 5 & \$ " & . \$ # 3 \* & \$ - / 2 3 # > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



( + , \* \$ + & \* \$ 8 9 \$ " & . \$ # 3 \* & \$ - / 2 3 # > \$

! " # \$ % & \$ ' \$ ( " ) \* \$



( + , \* \$. + 5 & # + \$ \* C / # \$ " & . \$ \* " # \$ ; 3 \* \* 0 \* > \$



D#"&. /&2E\$F +&. \*-/&2E

( +, \*\$A+-5"--. \$53\*&\*, \*-\$

G+\$5"66\$H\$&+#\$, /0/#\*. \$

( +, \*\$?"; <\$||||\$JKFL\$

! \*B\*B?\*-\$#3\*\$A++#9-/&#0\$

K! \$EE\$: \*M\*-L\$

GNOP\$9+00/?6\*\$B+, \*\$A-+B\$9-\* , /+80\$9+0/=+&\$

D#+- "2\*L\$

DP' QR\$

!



%#SO\$""\$TKGU\$6/A\*\$E

J+5\$#+\$98#\$"\*\$&\$. \$#+\$#3/0\$B/0\*-VL\$! %N\$

U+. \$?6\*00\$/#X\$

Y"B\*\$#/X\$

F 3\*&\*, \*-\$\*C/0#\$"\$9+00/?6\*\$B+, \*\$A-+B\$9-\* , /+80  
\$9+0/=+&0\$

F 3\*&\*, \*-\$#3\*\$0#" ; <\$0\$&+\$\*\$B9#V\$

2

2

1

0	1	0	0	1	1	0	1	1
1	0	0	1	0	0	1	1	1
0	1	1	0	1	1	1	0	1
1	1	0	0	1	0	0	1	0
1	0	0	1	0	1	1	0	1
0	0	1	1	0	1	0	1	1
0	1	0	0	1	1	0	0	0

8

1---

, 1 ≤  $\leq$  , 1 ≤  $\leq$  .

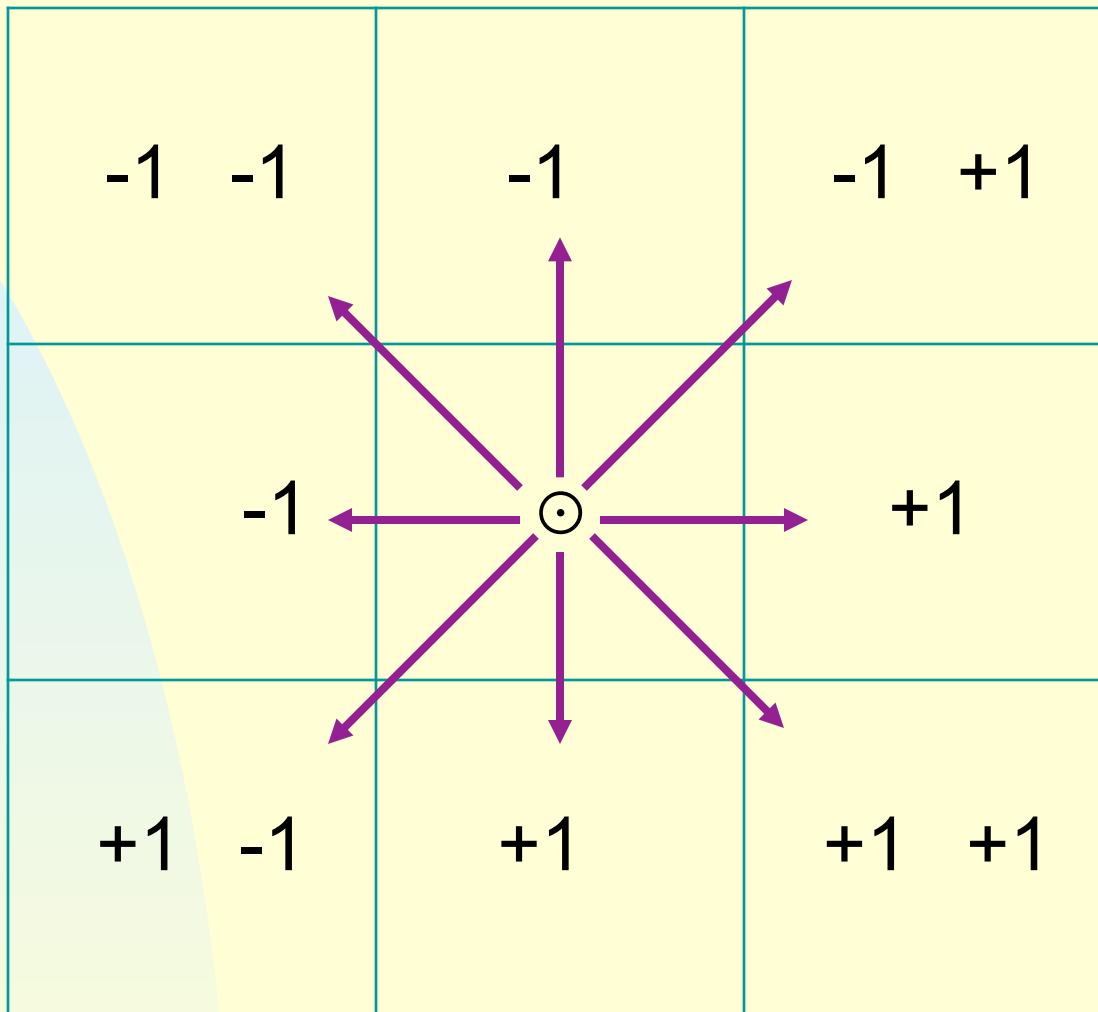
, 0 --- .

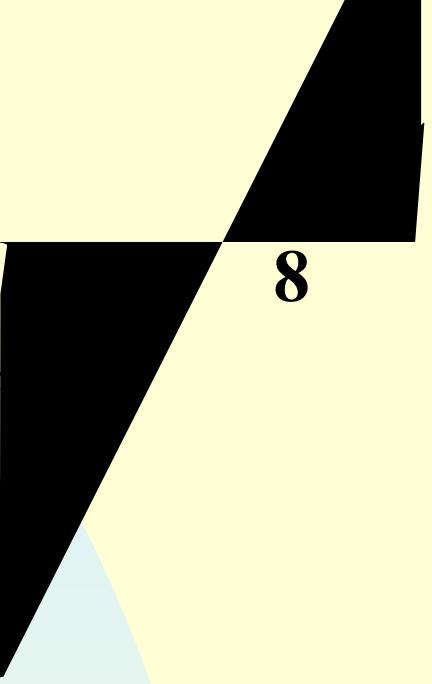
: 1 1 , : .

: .

1 , +2 +2 .

: , , , , , , .





```
8 :  
off  
a,b;  
;  
directions , , , , , , ;  
offsets mo e[8];
```

<b>q</b>	<b>mo e[q].a</b>	<b>mo e[q].b</b>
<b>N</b>	-1	0
<b>NE</b>	-1	1
<b>E</b>	0	1
<b>SE</b>	1	1
<b>S</b>	1	0
<b>SW</b>	1	-1
<b>W</b>	0	-1
<b>NW</b>	-1	-1

**Table of mo es**

,

+

+

• ;

• ;

:

# The basic idea:

8

,

,

.

,

+1

,

.

:

+2 +2 ,

0.

1

.

:

Initiali e stack to the ma e elementence coordinates and direction east;  
(stack is not empt )

(i, j, dir)=coordinates and direction from top of stack;  
pop the stack;  
(there are more mo es from (i, j))

(g, h)= coordinates of ne t mo e ;

((g=11k3 cm BT 24 (; ) Tj ]TJ ETW.2 (s) -0 0 0 1 0 (s) -06B

$((\text{!ma\_e}[g][h]) \&\& (\text{!mark}[g][h]))$  // legal and not visited

$\text{mark}[g][h]=1;$

$\text{dir}=\text{ne}$  t direction to tr ;

push (i, j, dir) to stack;

$(i, j, \text{dir}) = (g, h, N);$

<< No path in ma\_e. << ;



( m, p)

//Output a path (if an ) in the ma e; ma e[0][i] = ma e[m+1][i]

// = ma e[j][0] = ma e[j][p+1] = 1, 0 ≤ i ≤ p+1, 0 ≤ j ≤ m+1.

// start at (1,1)

mark[1][1]=1;

Stack<Items> stack(m\*p);

Items temp(1, 1, E);

stack.Push(temp);

(!stack.IsEmpty())

temp= stack.Top();

Stack.Pop();

i=temp. ; j=temp. ; d=temp.dir;

(d<8)

```
g=i+mo e[d].a;      h=j+mo e[d].b;  
((g==m) && (h==p)) // reached e it  
// output path  
<<stack;  
<< i<<   << j<<   << d<<       ; // last two  
<< m<<   << p<<       ;           // points  
;
```

```
((!ma_e[g][h]) && (!ma_e[g][h])) //ne position  
mark[g][h]=1;  
temp. =i; temp. =j; temp.dir=d+1;  
stack.Push(temp);  
i=g ; j=h ; d=N; // mo e to (g, h)
```

d++; // tr ne t direction

<< No path in ma e. << ;

```
    <<  
:  
    < T>  
ostream&           <<(ostream& os, Stack<T>& s)  
  
os << top= <<s.top<<      ;  
    (   i=0;i<=s.top;i++);  
os<<i<< : <<s.stack[i]<<      ;  
    os;  
  
    <<
```

•

ostream&

<<(ostream& os,Items& item)

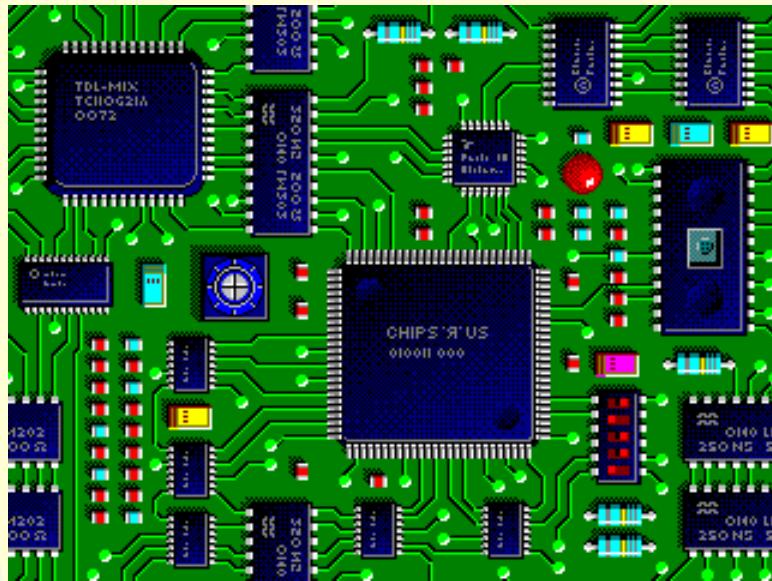
```
    os<<item. << , <<item. << , <<item.dir-1;  
// note item.dir is the next direction to go so the current  
// direction is item.dir-1.
```

,

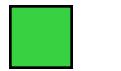
( \* ).

: **157-2, 3**





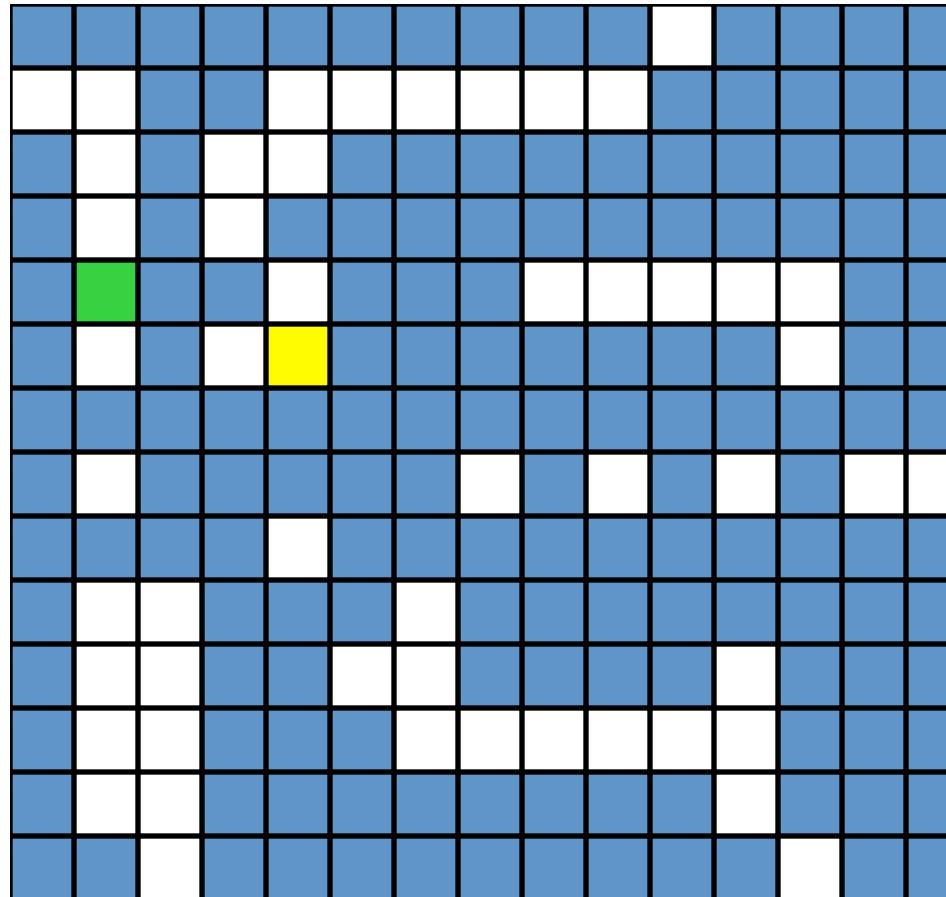
T\*\*\$0\$F/-\*\$! +8#\*-\$



start pin

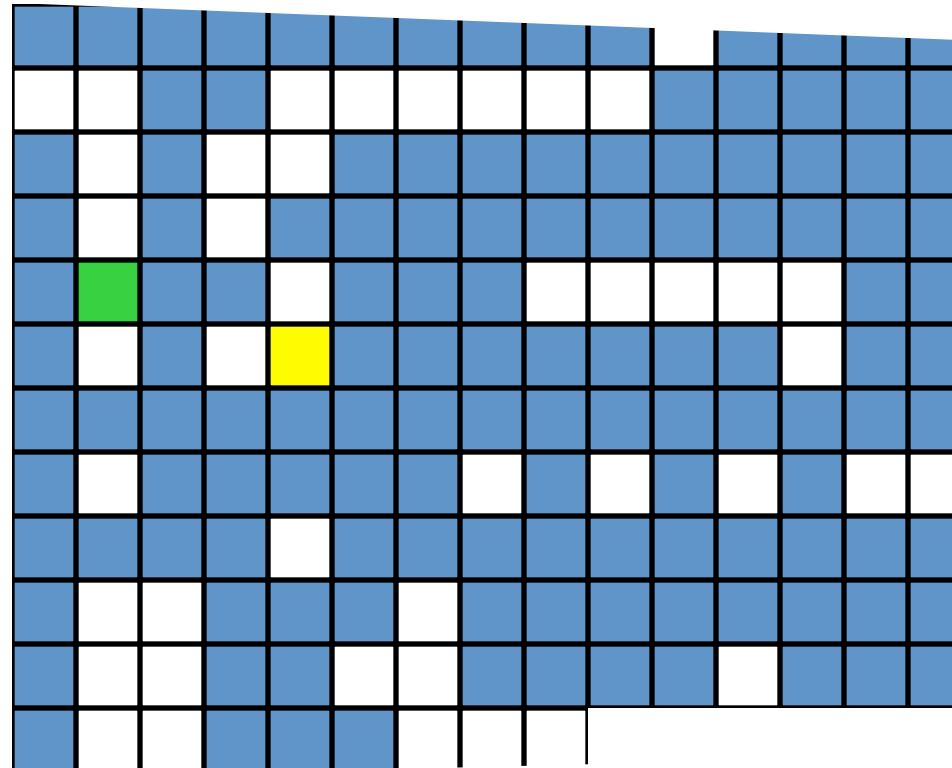


end pin



Label all reachable squares 1 unit from start.

T\*\*\$0\$F/-\*\$!+8#\*-\$



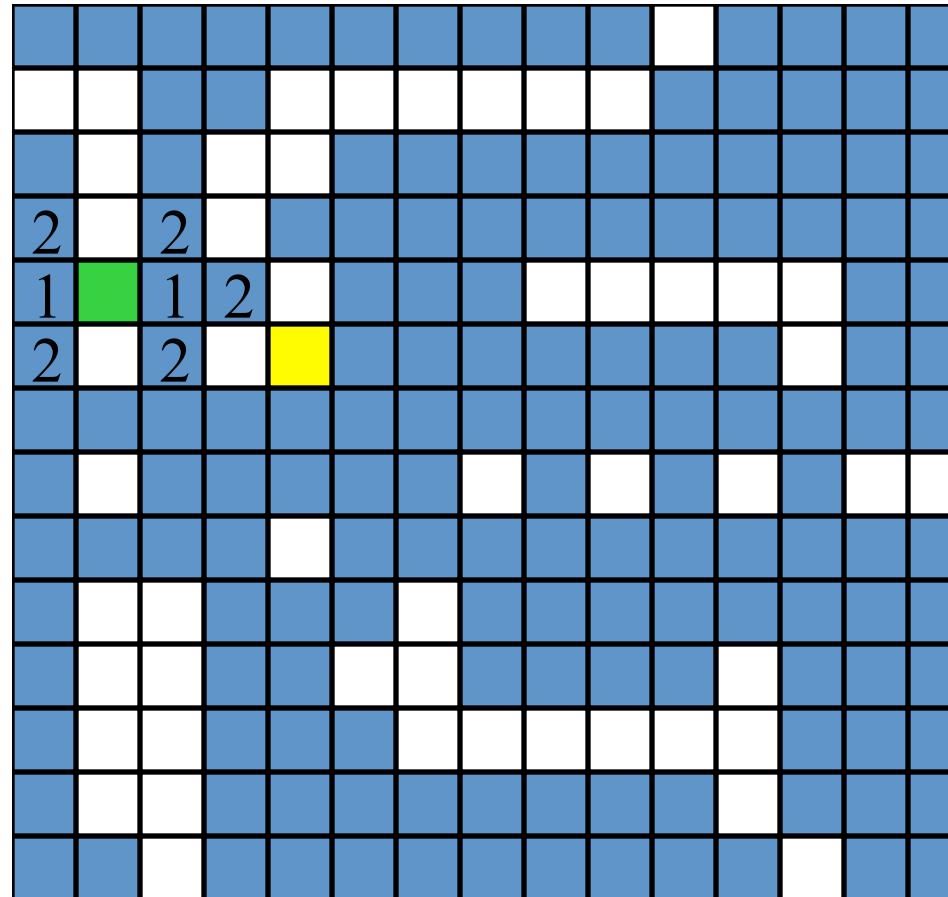
T\*\*\$0\$F/-\*\$! +8#\*-\$



start pin



end pin

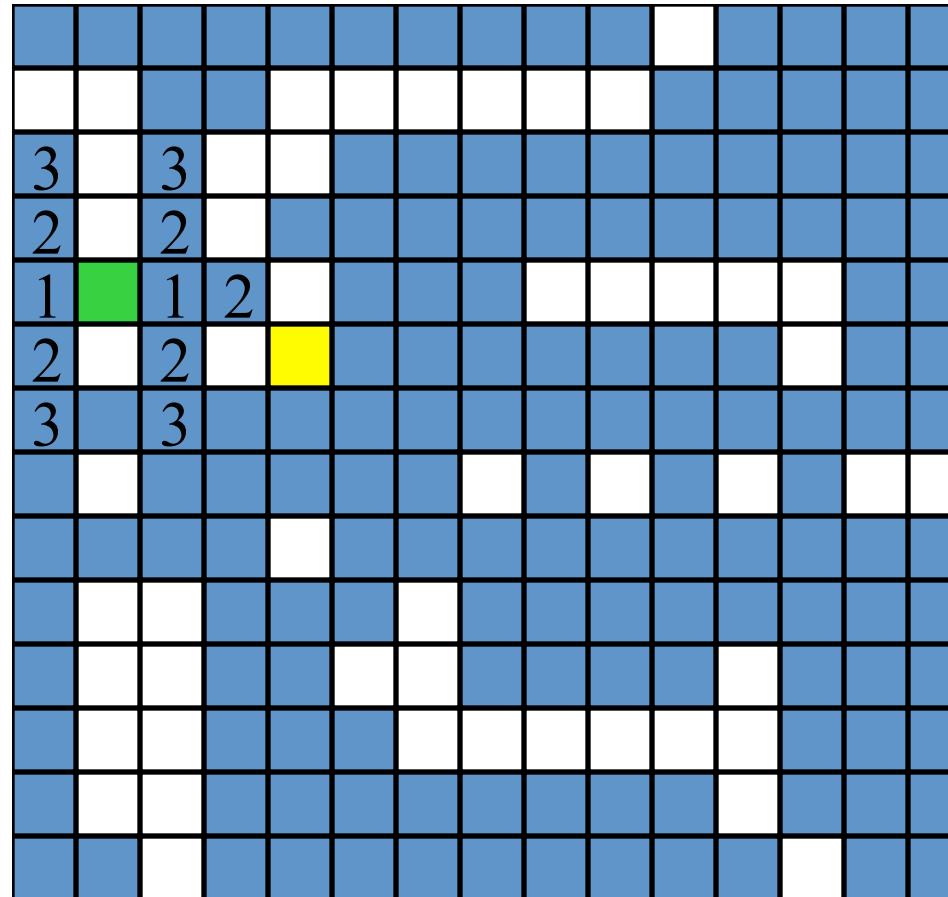


Label all reachable unlabeled squares 3 units from start.

T\*\*\$0\$F/-\*\$! +8#\*-\$

 start pin

 end pin

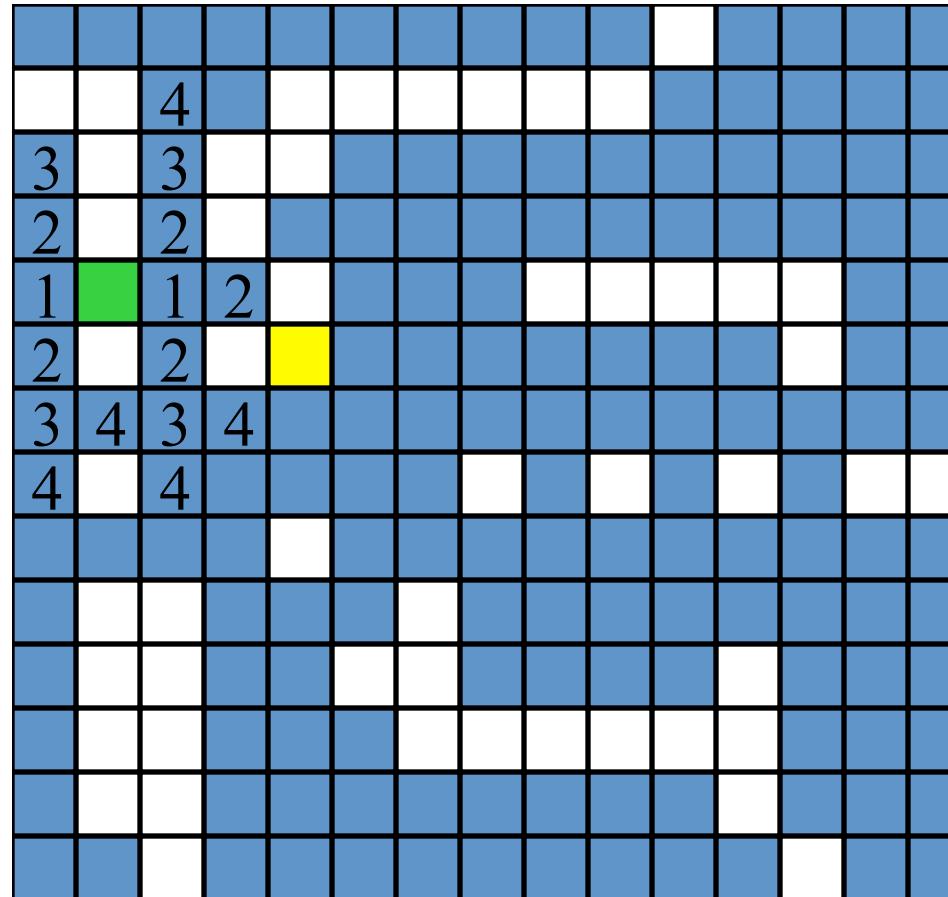


Label all reachable unlabeled squares 4 units from start.

T\*\*\$0\$F/-\*\$! +8#\*-\$

 start pin

 end pin

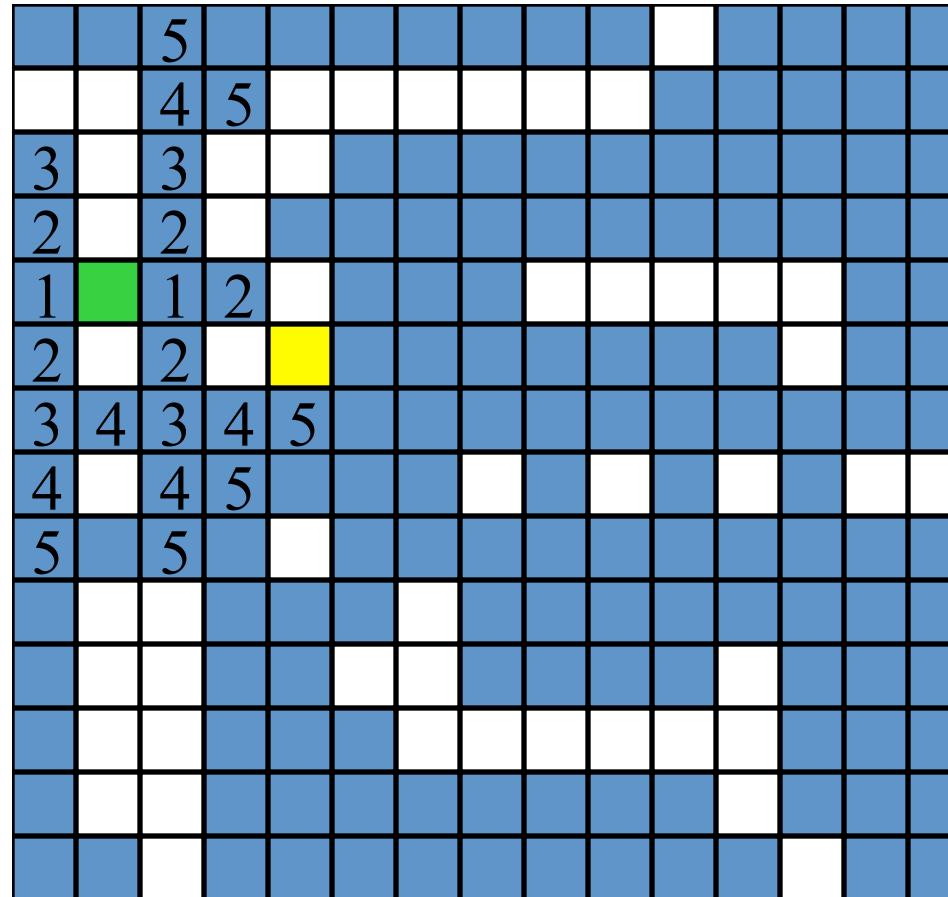


Label all reachable unlabeled squares 5 units from start.

T\*\*\$0\$F/-\*\$! +8#\*-\$

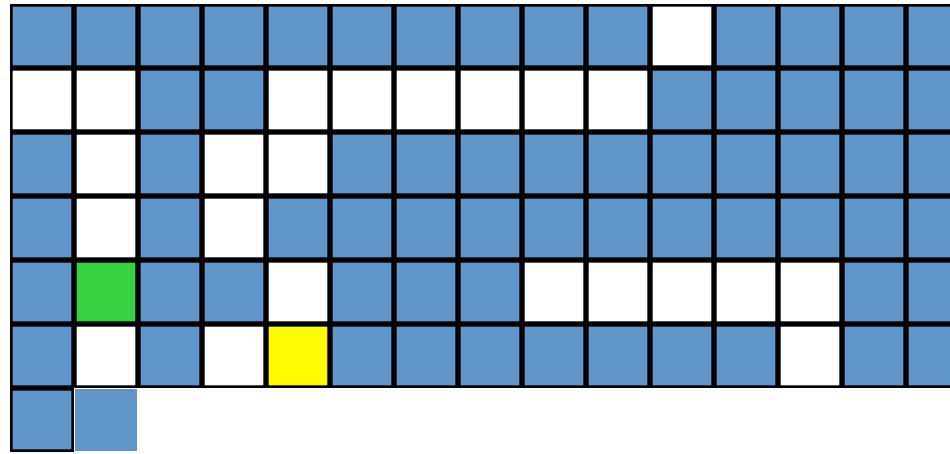
 start pin

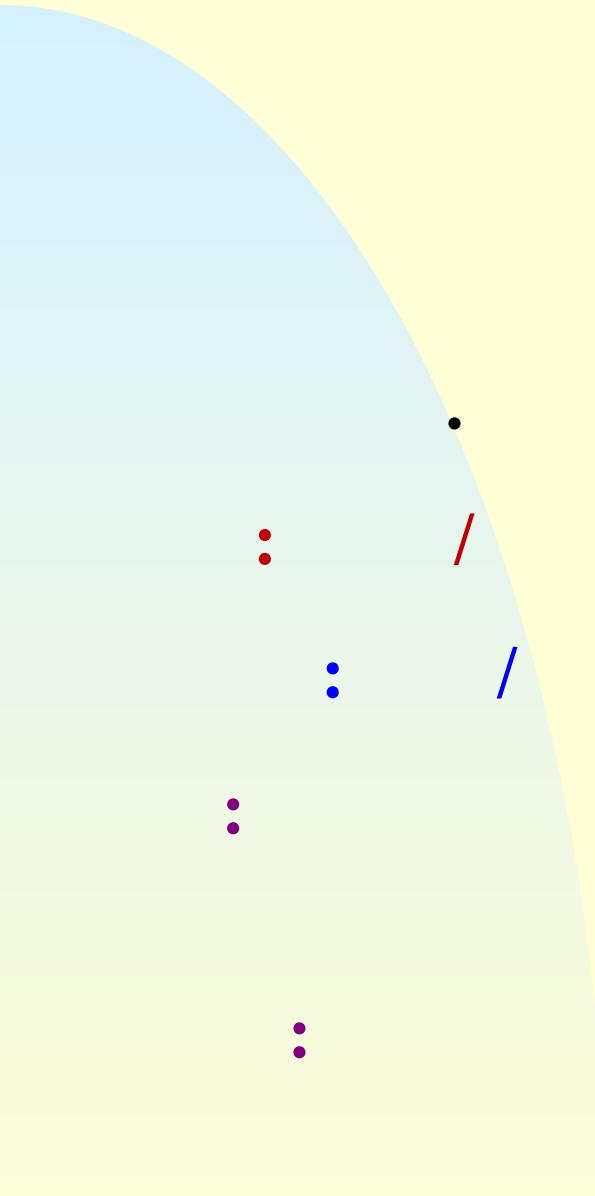
 end pin



Label all reachable unlabeled squares 6 units from start.

T\*\*\$0\$F/-\*\$!+8#\*-\$





• , ,  
• ,  
: / + \* \*  
: / \* + \*  
: - ( ) .  
:

++.

•

•

,

•

,

•

•

---

1

, !

2

\*, /, %

3

+, -

4

, , , ,

5

, !

6

&&

7

---

**Problem:**

**how to evaluate an expression?**

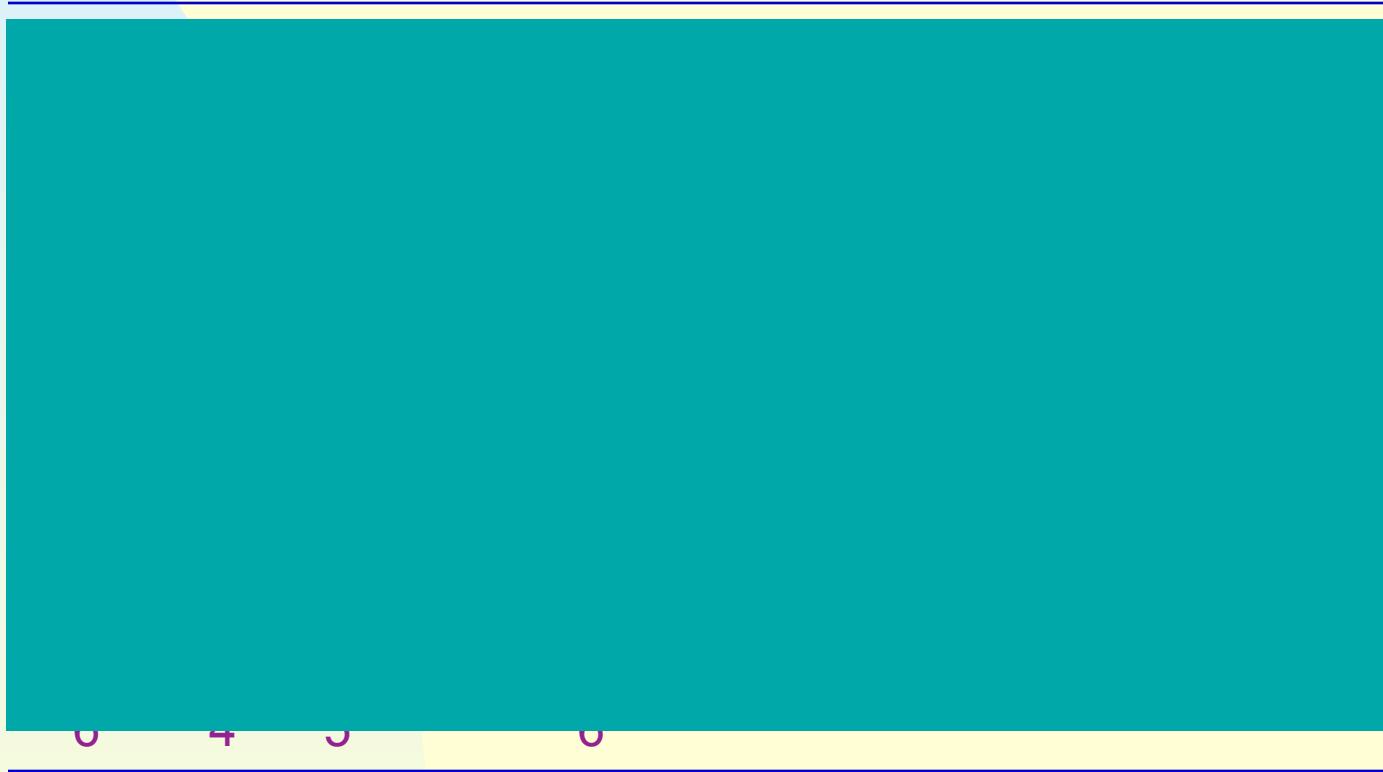
: / \* + \*

,

, ≥1.

:

**A B / C   D E \* + A C \***



**T<sub>6</sub> is the res It.**

•

•

✓

✓

✓



E eval(E expression e,

```
// evaluate expression e. It is assumed that the  
// last token is #. A function Ne tToken is used to get  
// the next token from e. Use stack.
```

```
Stack<Token> stack; //initialise stack
```

```
(Token token = Ne tToken(e); != #; =Ne tToken(e))
```

```
( is an operand) stack.Push( );
```

```
    // operator
```

```
remove the correct number of operands for operator  
from stack; perform the operation and store the result  
(if any) onto the stack;
```



## Infi to Postfi

Idea: note the order of the operands in both infi and postfi

infi : A / B C + D \* E A \* C

postfi : A B / C D E \* + A C \*

the same!

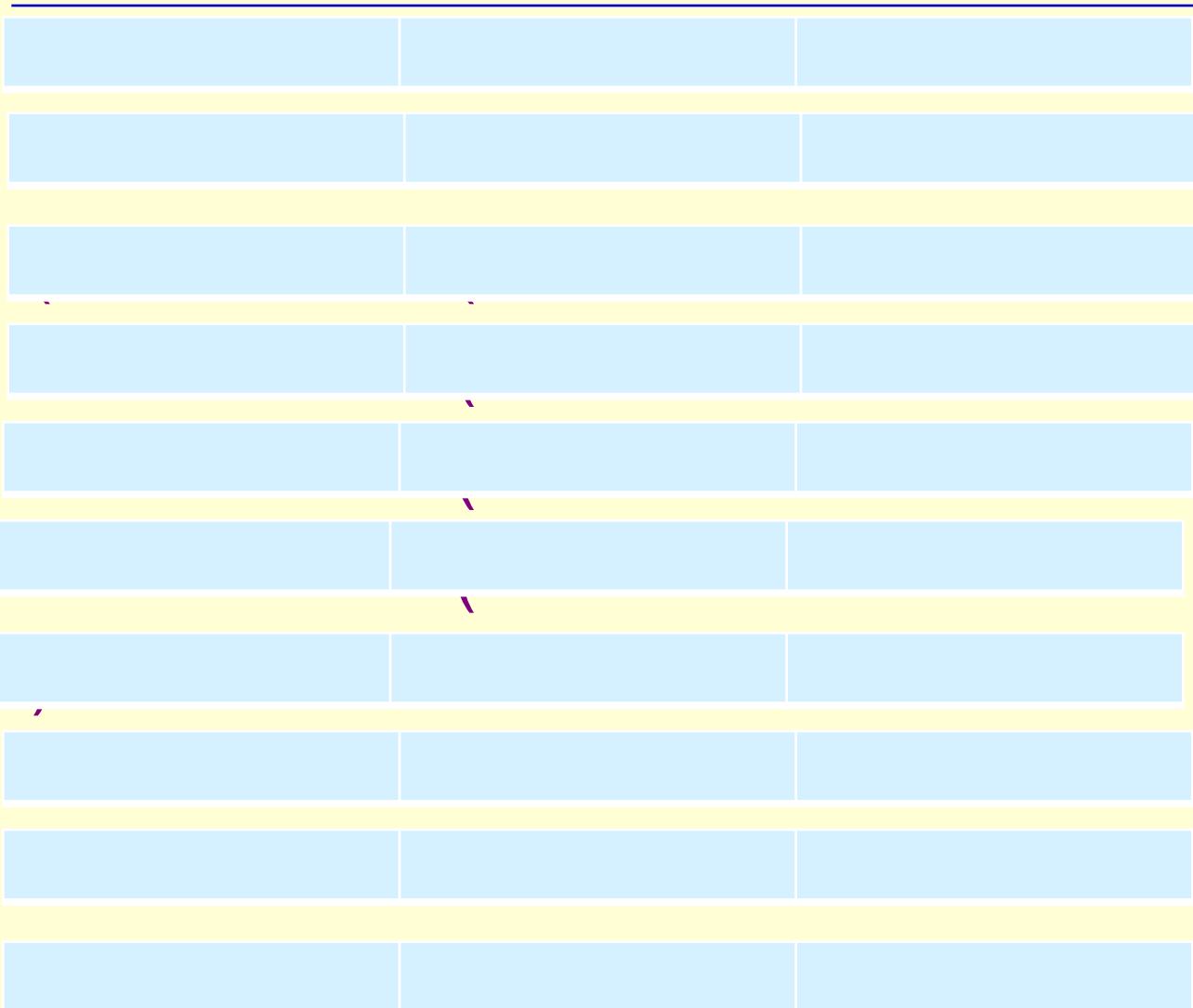
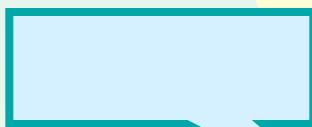
immediatel passing an operands to the o tp t  
store the operators in a **stack** ntil the right time.

e.g.

A\*(B+C)\*D → ABC+\*D\*

$A^*(B+C)^*D$

$\rightarrow ABC+^*D^*$



- ,
- ,
- .
- ( - )
- ( - )
- . 3.15
- ( ( ) 8,    ( ( ) 0,    ( # ) 8

Hence the rule:

**Operators are taken out of stack as long as their isp is numerically less than or equal to the icp of the next operator.**

Postfi (E presson e)

```
// output the postfi of the infi e pression e. It is assumed  
// that the last token in e is # . Also, # is used at the bottom  
// of the stack.
```

```
Stack<Token> stack; //initiali e stack
```

```
stack.Push( # );
```

```
(Token =Ne tToken(e); != # ; =Ne tToken(e))
( is an operand)    << ;
( == ) )
// unstack until (
( ; stackTop() != ( ; stack.Pop())
<< stack.Top();
stack.Pop(); // unstack (
// is an operator
( ; isp(stack.Top()) <= icp( ); stack.Pop())
<< stack.Top();
stack.Push( );
// end of expression, empt the stack
( ; !stack.IsEmpty()); << stack.Top(), stack.Pop());
<< ;
```

- :
- :
- , ( ).
- .

1 ( # ) +

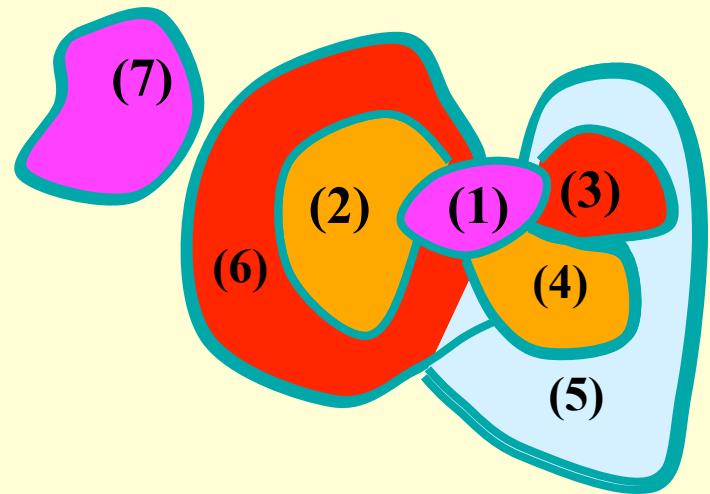
## Ercises: P165-1,2

Can e e al ate infi e pressions directl ?

infi : A / B C + D \* E A \* C

7 7

	1	2	3	4	5	6	7
1	0	1	1	1	1	1	0
2	1	0	0	0	0	1	0
3	1	0	0	1	1	0	0
4	1	0	1	0	1	1	0
5	1	0	1	1	0	1	0
6	1	1	0	1	1	0	0
7	0	0	0	0	0	0	0



1	2	3	4	5	6	7
1	2	2				1

0 1;  
1 ;  
( < )  
(( )) && ( > ))  
0 ;  
( < ) && ( \*  
// 判  
( < ) ++ ; // +1

