Decision Making of Networked Multiagent Systems for Interaction Structures

Yichuan Jiang, *Member, IEEE*, Jing Hu, and Donghui Lin

***Abstract*—Networked multiagent systems are very popular in large-scale application environments. In networked multiagent systems, the interaction structures can be shaped into the form of networks where each agent occupies a position that is determined by such agent's relations with others. To avoid collisions between agents, the decision of each agent's strategies should match its own interaction position, so that the strategies available to all agents are in line with their interaction structures. Therefore, this paper presents a novel decision-making model for networked multiagent strategies based on their interaction structures, where the set of strategies for an agent is conditionally decided by other agents within its dependence interaction substructure. With the presented model, the resulting strategies available to all agents can minimize the collisions of multiagents regarding their interaction structures, and the model can produce the same resulting strategies for the isomorphic interaction structures. Furthermore, this paper uses a multiagent citation network as a case study to demonstrate the effectiveness of the presented decision-making model.**

***Index Terms*—Citation networks, decision making, multiagents, networked interaction structures, social network analyses.**

structures. In our decision-making model, we focus upon the multiagent dependence relations that are always seen where the strategies of some agents are dependent on others [12]. Dependence relations occur in multiagent systems due to many reasons, such as resource constraints, environmental constraints, task allocations, etc. Clearly, the strategies available to an agent should be conditionally determined by its dependenta]TJ0 -128.pendenceshoiueny

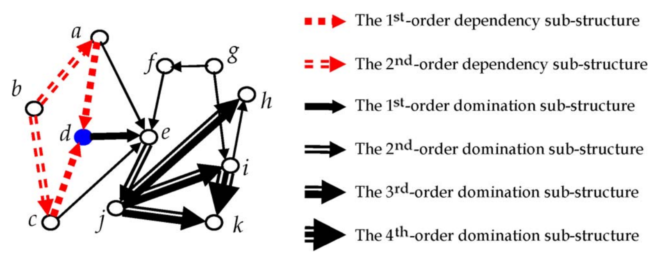


Fig. 2. Dependence and domination substructures of agent d.

Obviously, *the second-order domination substructure of* a_i can be defined as

$$Dom\left(Dom_{a_i}\right) = \{\langle a_j, a_k\rangle | a_j \in A \wedge a_k \in A \wedge \langle a_i, a_j\rangle \in R \wedge \langle a_j, a_k\rangle \in R\}. \quad (8)$$

Therefore, *the* n*th-order domination substructure of* a_i can be defined as

$$\prod_n Dom_{a_i} = \overbrace{Dom\left(Dom\left(\cdots\left(Dom_{a_i}\right)\cdots\right)\right)}^{n} = \{\langle a_{n-1}, a_n\rangle | a_1 \in A \wedge a_2 \in A \wedge \cdots \wedge a_n \in A \wedge \langle a_i, a_1\rangle \in R \wedge \langle a_1, a_2\rangle \in R \wedge \cdots \wedge \langle a_{n-1}, a_n\rangle \in R\}. \quad (9)$$

The set of agents within the first-order domination substructure of a_i (called *its first-order domination agents*) is

$$\Omega_{a_i} = \{a_j | a_j \in A \wedge \langle a_i, a_j\rangle \in Dom_{a_i}\} = \{a_j | a_j \otimes r \wedge r \in Dom_{a_i}\}. \quad (10)$$

Therefore, the set of all agents within the all-orders domination substructures of agent a_i is

$$\sum \Omega_{a_i} = \bigcup_k \left\{a_j | a_j \otimes r \wedge r \in \prod_k Dom_{a_i}\right\}. \quad (11)$$

Example 2: Now, we consider the dependence and domination substructures of agent d in Fig. 1, as shown in Fig. 2

$$Dep_d = \{\langle a, d\rangle, \langle c, d\rangle\} \quad \prod_2 Dep_d = \{\langle b, a\rangle, \langle b, c\rangle\}$$

$$\mho_d = \{a, c\} \quad \sum \mho_d = \{a, c, b\} \quad Dom_d = \{\langle d, e\rangle\}$$

$$\prod_2 Dom_d = \{\langle e, j\rangle\} \quad \prod_3 Dom_d = \{\langle j, h\rangle, \langle j, i\rangle, \langle j, k\rangle\}$$

$$\prod_4 Dom_d = \{\langle i, k\rangle\} \quad \Omega_d = \{e\}, \sum \Omega_{a_i} = \{e, j, h, i, k\}.$$

Lemma 1: Let an agent interaction structure be $N = \langle A, R\rangle$. If N is a directed acyclic graph (DAG), we have $\forall a, b \in A$, $a \in \sum \Omega_b \Rightarrow a \notin \sum \mho_b$ and $a \in \sum \mho_b \Rightarrow a \notin \sum \Omega_b$.

Proof:

1) If $\exists a, b \in A \Rightarrow a \in \sum \Omega_b \wedge a \in \sum \mho_b$, $a \in \sum \Omega_b$ denotes that there is a path from b to a and $a \in \sum \mho_b$ denotes that there is a path from a to b; therefore, there is a cycle which contains a and b.
2) If $\exists a, b \in A \Rightarrow a \in \sum \mho_b \wedge a \in \sum \Omega_b$, $a \in \sum \mho_b$ denotes that there is a path from a to b and $a \in \sum \Omega_b$ denotes that there is a path from b to a; hence, there is a cycle that contains a and b.

Obviously, those situations are impossible in a DAG; therefore, we have Lemma 1. □

If two agents have identical ties to and from all other agents in the interaction structure, we can say that they are structurally equivalent.

Definition 4: Interaction structural equivalence[8]. Let the agent interaction structure be $N = \langle A, R\rangle$, where A denotes the set of agents and R denotes the set of agent interaction relations; $|A| = m$, $|R| = n$, $a_i, a_j \in A$, and $1 \leq i, j \leq m$. Then, a_i and a_j are structurally equivalent if for all agents $a_k \in A$, $k = 1, \ldots, m$ and $k \neq i, j$, and all interaction relations r_x, $x = 1, \ldots, n$, a_i has an interaction relation to a_k, if and only if a_j also has an interaction relation to a_k, and a_i has an interaction relation from a_k, if and only if a_j also has an interaction relation from a_k. If a_i and a_j are structurally equivalent, we can denote them as $a_i \equiv a_j$.

Lemma 2: If two agents are structurally equivalent, then they have the same first-order dependence and domination agents, i.e., $(a_i \equiv a_j) \Rightarrow (\mho_{a_i} = \mho_{a_j}) \wedge (\Omega_{a_i} = \Omega_{a_j})$. Moreover, they also have the same nth-order $(n > 1)$ dependence and domination agents.

Proof: From Definition 4, if two agents are structurally equivalent, they have the same first-order dependence agents and first-order domination agents. According to (4) and (9), the nth-order $(n > 1)$ dependence substructure is fully dependent on the $(n-1)$th-order dependence agents and the nth-order $(n > 1)$ domination substructure is fully controlled by the $(n-1)$th-order dependence agents. Therefore, the two agents have the same nth-order $(n > 1)$ dependence and domination agents. □

Example 3: From Fig. 1, the agent sets that have immediate *in* interaction relations to a and c are the same, namely, $\{b\}$, and the agent sets that have immediate *out* interaction relations from a and c are also the same: $\{d, e\}$. Therefore, agents a and c are structurally equivalent. Moreover, h and k are also structurally equivalent. Obviously, $\mho_a = \mho_c = \{b\}$, $\Omega_a = \Omega_c = \{d, e\}$, $\mho_h = \mho_k = \{i, j\}$, $\Omega_h = \Omega_k = \{\ \}$, $\sum \mho_a = \sum \mho_c = \{b\}$, $\sum \Omega_a = \sum \Omega_c = \{d, e, j, h, i, k\}$, and $\sum \mho_h = \sum \mho_k = \{i, j, g, e, a, c, d, b, f\}$; therefore, Lemma 2 is validated.

Definition 5: If agent a is not in the all-orders dependence and domination structures of agent b, i.e., $(a \notin \sum \mho_b) \wedge (a \notin \sum \Omega_b)$ is true, then we can think that agent a is independent from agent b, which can be denoted as $a \not\triangleright b$. Obviously, we have $\forall a, b \in A$, $a \not\triangleright b \Rightarrow b \not\triangleright a$.

Lemma 3: Let an agent interaction structure be $N = \langle A, R\rangle$; if N is a DAG, we have $\forall a, b \in A$, $a \equiv b \Rightarrow a \not\triangleright b$.

Proof: Let $\exists a, b \in A$, $a \equiv b$. If $\neg(a \not\triangleright b)$ is true, then $(a \in \sum \Omega_b) \vee (b \in \sum \Omega_a)$ is true. Now, $a \equiv b$; hence, $(\sum \mho_a = \sum \mho_b) \wedge (\sum \Omega_a = \sum \Omega_b)$ is true, which denotes that $(a \in \sum \Omega_a) \vee (b \in \sum \Omega_b)$ is true. Such situation is impossible in a

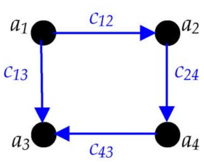
B. Constraints Among Agents' Strategies in the Interaction Structures

When agents interact with and depend on each other, there may be some constraints which limit their available strategies for avoiding collisions. For example, if there are two paths between places x and y and each path can only be passed by one agent at the same time, agent a_1 will go from x to y and agent a_2 will go from y to x. Moreover, a_1 has the priority to select the path first (i.e., the decision of strategy of a_2 depends on the strategy of a_1). Thus, we can set the constraint to a_2 as "a_2 *cannot select the same path as* a_1".

Definition 6: Social constraint. Let there be, first, a finite set of agents, $A = \{a_1, a_2, \ldots, a_n\}$, and second, an initial set of strategies for each agent, containing a finite and discrete strategies' domain for each agent, $S = \{S_1, S_2, \ldots, S_n\} \forall i \in [1, n]$ and $s_{ij} \in S_i$, where s_{ij} is the jth strategy that agent a_i adopts in the operation. Then, a social constraint set is $C = \{C(A_1), C(A_2), \ldots, C(A_m)\}$, where each A_i is a subset of the agents and each social constraint $C(A_i)$ is a set of tuples indicating the mutually consistent strategy values of the agents in A_i.

In reality, the social constraint with the arity of 2 is always seen and is the basic form of most constraints. Thus, we mainly consider such constraint form in this paper.

Definition 7: A binary social constraint is the one that only affects two agents. If there is a binary social constraint c_{ij} and535.4(fromnd533hen,)-450.2(agent)]TJ9.5 1 Tf1.0434 0 TD0 Tc(a)Tj/F8 1 Tf6.9738 0 10 98 TD3.7(o03



$c_{12} : s_1 \geq s_2;$
$c_{24} : (s_2 - 2) > s_4;$
$c_{13} : (s_1 - 5) > s_3;$
$c_{43} : s_4 > s_3.$

(a)

Strategy profiles of agents				Social constraints				U
a_1	a_2	a_3	a_4	c_{12}	c_{24}	c_{13}	c_{43}	
{1,…,10} f_1=1	{1,…,10} f_2=1	{1,…,10} f_3=1	{1,…,10} f_4=1	√	×	×	×	[illegible]/8
{1,3,5,7} f_1=0.4	{1,3,5,7} f_2=0.4	{1,3,5,6} f_2=0.4	{1,3,4} f_2=0.3	√	√	×	×	$\frac{7}{16}$
{5,6,8,9} f_1=0.4	{3,5,10} f_1=0.3	{2,3,5,7} f_1=0.4	{1,3} f_1=0.2	×	√	×	×	$\frac{23}{80}$
{1,…,10} f_1=1	{1,…,9} f_1=0.9	{1, …,4} f_1=0.4	{1, …,6} f_1=0.4	√	√	√	√	$\frac{67}{80}$

Remarks: α=0.5, β=0.5. √: lawful, ×: unlawful. *U*: usefulness degree.

(b)

Fig. 3. Example for strategy constraints and decision making.

the usefulness degree of a decision SL as $|C_{\text{sat}}|/|C|$. Moreover, a decision should enable the agents to have more total freedom under the condition that social constraints can be satisfied. Thus, we can extend the definition as

$$U_{\text{SL}} = \alpha \left(\left(\sum_{i=1}^{n} f_i \right) /n \right) + \beta \left(|C_{\text{sat}}|/|C| \right) \qquad (13)$$

where α and β are parameters to define the relative importance of agent freedom degree and dependence constraint satisfaction degree, respectively. We can set the values of the parameters according to real situations. Therefore, our aim of decision making is to explore the decision with the maximum degree of usefulness.

Example 4: Fig. 3 is an example of the decided multiagent strategies with different usefulness degrees. Fig. 3(a) is an interaction structure where there are four agents $\{a_1, a_2, a_3, a_4\}$ and four social constraints taken by the dependence relations $\{c_{12} : s_1 > s_2, c_{24} : (s_2 - 2) > s_4, c_{13} : (s_1 - 5) > s_3, c_{43} : s_4 > s_3\}$. In such a system, it is assumed that the agents can take actions of adopting values in the set $S = \{1, 2, \ldots, 10\}$; hence, the strategies are the values in S. The initial strategy profiles of four agents are all set to {1, 2, …, 10}. Now, we randomly make four decisions and compute their usefulness degrees, as shown in Fig. 3(b).

B. Basic Decision-Making Model for DAG Structures

If an interaction structure $N = \langle A, R \rangle$ is a DAG, then we have the following: $\forall a_i \in A$, $\mho_{a_i}$ is fixed [16]. The basic idea in our model is as follows: $\forall a_i \in A$, if we want to decide the set of available strategies for agent a_i, we should decide the sets of available strategies for $\forall a_i \in \mho_{a_i}$ in advance. Thus, we can obtain the joint distribution of strategies for all agents step by step.

Therefore, our algorithm can be designed as follows: At first, we restrict the available strategies of the agents whose dependence agents are all decided or empty; such iteration will be repeated until it cannot find any undecided agents whose dependence ones are all decided or empty. Now, if all agents in the system can be decided with definite strategies, then the decision making is successful; otherwise, it can be noted that there are cycles in the interaction structure.

Algorithm 1. Decision making of multiagent strategies for directed acyclic interaction structure

- *Input* $A = \{a_1, a_2, \ldots, a_n\}$ *and* R;
- *Input* $S = \{S_1, S_2, \ldots, S_n\}$; /∗ the initial strategies ∗/
- *Creatstack (stack)*;
- *For* $\forall a_i \in A$:
  - *if* $\mho_{a_i} = \{\ \}$, *push* (a_i, stack);
- $A' = \{\ \}$;
- *While* (!empty(stack)) *do*:
  1) $a_u = pop$(stack);
  2) $A' = A' \cup \{a_u\}$;
  3) *for* agent $\forall a_j \in \Omega_{a_u}$ *do*:
     - i) *Restrict* S_j *according to* c_{uj};
     - ii) $\mho_{a_j} = \mho_{a_j} - \{a_u\}$;
     - iii) *if* $\mho_{a_j} = \{\ \}$, *push* (a_j, stack);
- If $A == A'$, return ("There are no cycles");
  - *else* return ("There are cycles");
- *Output* S_{a_i} $\forall a_i \in A'$.

Algorithm 1 is $O(n^*e)$, where n denotes the number of agents and e denotes the number of dependence relations.

Theorem 1: Let the interaction structure be $N = \langle A, R \rangle$. If N is a DAG, then Algorithm 1 can make a unique decision, i.e., the decided strategies for all agents are definite.

Proof: $\forall a_i \in A$, the set of available strategies for a_i in the decision making is fully determined by the following three factors:

1) the initial set of strategies for a_i;
2) the set of social constraints using a_i as object agent, namely, Dep_{ai};
3) the set of available strategies for $\mho_{a_i}$ in the decision making.

Obviously, the uniqueness of 1) can be satisfied. Now, the core procedure of Algorithm 1 is the same as the one of the Topology Sorting Algorithm [17]; hence, $\mho_{a_i}$ is unique and the uniqueness of 2) and 3) can also be satisfied. Therefore, Algorithm 1 can make a unique decision for a directed acyclic structure. □

Lemma 4: In the multiagent strategies decided by Algorithm 1, if the set of strategies for one agent is empty, then the dependence relations using such agent as object are unlawful.

Proof: From Definition 8, we have Lemma 4. □

Two graphs containing the same number of graph vertices connected in the same way are considered isomorphic [18], [19]; now, we present the definition of isomorphic multiagent interaction structures, shown as follows.

Definition 14: Let there be two interaction structures; one is G with the agent set $A_g = \{a_{g1}, \ldots, a_{gn}\}$, and the other is H with the agent set $A_h = \{a_{h1}, \ldots, a_{hn}\}$. G and H are said to be *isomorphic* if, first, there is a bijection f such that interaction

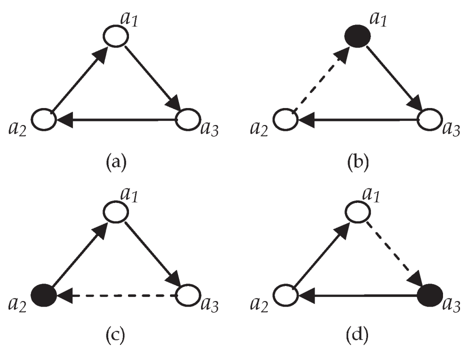


Fig. 4. Directed cyclic dependence structure.

relation $\langle a_i, a_j \rangle$ is in G iff $\langle f(a_i), f(a_j) \rangle$ is in H, and second, the constraints taken by the interaction relations $\langle a_i, a_j \rangle$ and $\langle f(a_i), f(a_j) \rangle$ are the same.

Theorem 2: Assume a scenario in which two interaction structures G and H satisfy the following: 1) G and H are both DAGs; 2) G and H are isomorphic; and 3) $\forall as_i \in As$ and its peer in H, namely, $f(as_i)$, the initial strategies of as_i and $f(as_i)$ are the same. Then, we can deduce that the decided strategies of as_i and $f(as_i)$ by using Algorithm 1 are the same.

Proof: From the definitions of conditional strategy and decision making in the interaction structure, for agent a, it is determined by its first-order dependence structure Dep_a. Therefore, a's final strategies in the restriction of decision making is determined by the following: 1) the agents in Dep_a (i.e., $\mho_a$); 2) the strategies of $\mho_a$; and 3) the constraints taken by the interaction relations from $\forall a_j \in \mho_a$ to a. Now, while Algorithm 1 is used, as_i and $f(as_i)$ have the same three factors if G and H are isomorphic; thus, the decided strategies of as_i and $f(as_i)$ are the same. □

C. Extended Decision-Making Model for Interaction Structures With Cycles

If there are any cycles in the interaction structures, then there exist some agents whose $\mho_{a_i}$ cannot be decided. For example, Fig. 4(a) is a directed cyclic structure, where $a_1 \in \mho_{a_3}$, $a_3 \in \mho_{a_2}$, and $a_2 \in \mho_{a_1}$. Therefore, we cannot make a definite decision according to Algorithm 1.

There are many forms of cycles; among them, the simple cy-
i

- *Calling Algorithm 1*; /∗ After the execution of Algorithm 1, now, the set of remaining agents is $(\cup_i A_i'') \cup A^*$ ∗/
- *For each*

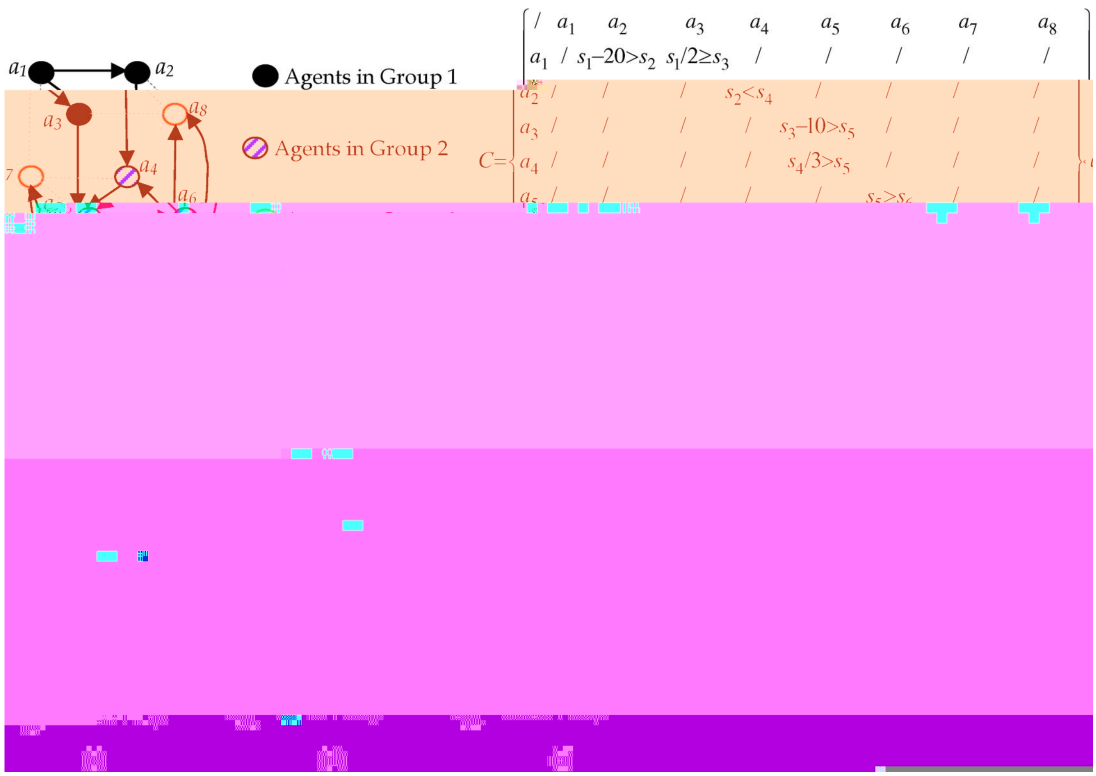


Fig. 6. Case demonstration for Algorithm 3. (a) Interaction structure and initial strategies. (b) Social constraints of dependence relations. (c) Strategy decision result by selecting a_4 to break the cycle. (d) Strategy decision result by selecting a_5 to break the cycle. (e) Strategy decision result by selecting a_6 to break the cycle.

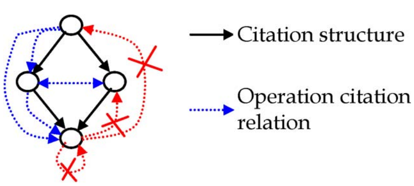


Fig. 7. Example for the citation network and operation citation relations.

4) An agent cannot cite the operation result of itself (*Citation Rule 4*).
5) Agent a cannot cite the results of agents in the all-orders dependence structures of a (*Citation Rule 5*).
6) If two agents are independent of each other in the interaction structures, then they can cite each other (*Citation Rule 6*).

Example 6: Fig. 7 shows an agent citation network; therefore, now, we can design some operation citation relations.

B. Strategies and Decision Making in Multiagent Citation Networks

In citation networks, the strategies of a are the set of agents from which a can cite operation results. For example, if $S_a = \{a_1, a_2, a_3\}$, then agent a can cite the operation results from agents a_1, a_2, and a_3.

Definition 15: In the environment of citation network $\langle A, R\rangle$, where A denotes the agents and R denotes the citation links among agents, *a useful decision* is the one that restricts the citation relations among agents to satisfy the requirements of multiagent citation rules.

Obviously, to satisfy the requirements of citation rules $\forall a \in A$, the set of strategies of agent a should have the following properties (**decision laws**).

1) Law 1: $\forall a \in A, \sum \Omega_a \subseteq S_a$ (Citation Rule 1 and 2).
2) Law 2: $\forall a, b \in A, b \in \sum \mho_a \Rightarrow b \notin S_a$ (Citation Rules 3 and 5).
3) Law 3: $\forall a \in A, a \notin S_a$ (Citation Rule 4).
4) Law 4: $\forall a, b \in A,\ a \not\triangleright b \Rightarrow (a \in S_b \wedge b \in S_a)$ (Citation Rule 6).

Now, according to the decision laws, we can design the conditional strategy set of agent $a \in A$ as

$$S_{a|\mho_a} = A - \{a\} - \sum \mho_a. \tag{14}$$

Therefore, the decision of the whole system can be the joint distribution of the conditional strategies of all agents which can satisfy the requirements of citation rules, i.e., we have

$$SL = S(a_1, a_2, \cdots a_n) = \overset{n}{\underset{i}{\wedge}} S_{a_i|\mho_{a_i}} = \overset{n}{\underset{i}{\wedge}} \left(A - \{a_i\} - \sum \mho_{a_i}\right). \tag{15}$$

Theorem 5: The decision making of multiagent strategies implemented by (14) and (15) is useful, and the final set of strategies available to all agents can satisfy the citation rules (i.e., can satisfy the decision laws).

Proof: Now, we prove that the decided strategies satisfy the four decision laws.

1) $\forall b \in \sum \Omega_a \Rightarrow b \notin \sum \mho_a$, we have $b \in (S_{a|\mho_a} = A - \{a\} - \sum \mho_a)$ according to (14). Therefore, in the

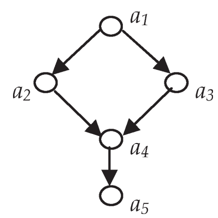


Fig. 8. Example to demonstrate the conditional strategy and decision making in citation networks.

decision results determined by (15), a can cite operation results from agent $b \forall b \in \sum \Omega_a$, which then satisfies Law 1.

2) For agent $a \in A$, if b is the all-orders dependence agents of a, i.e., $b \in \sum \mho_a$

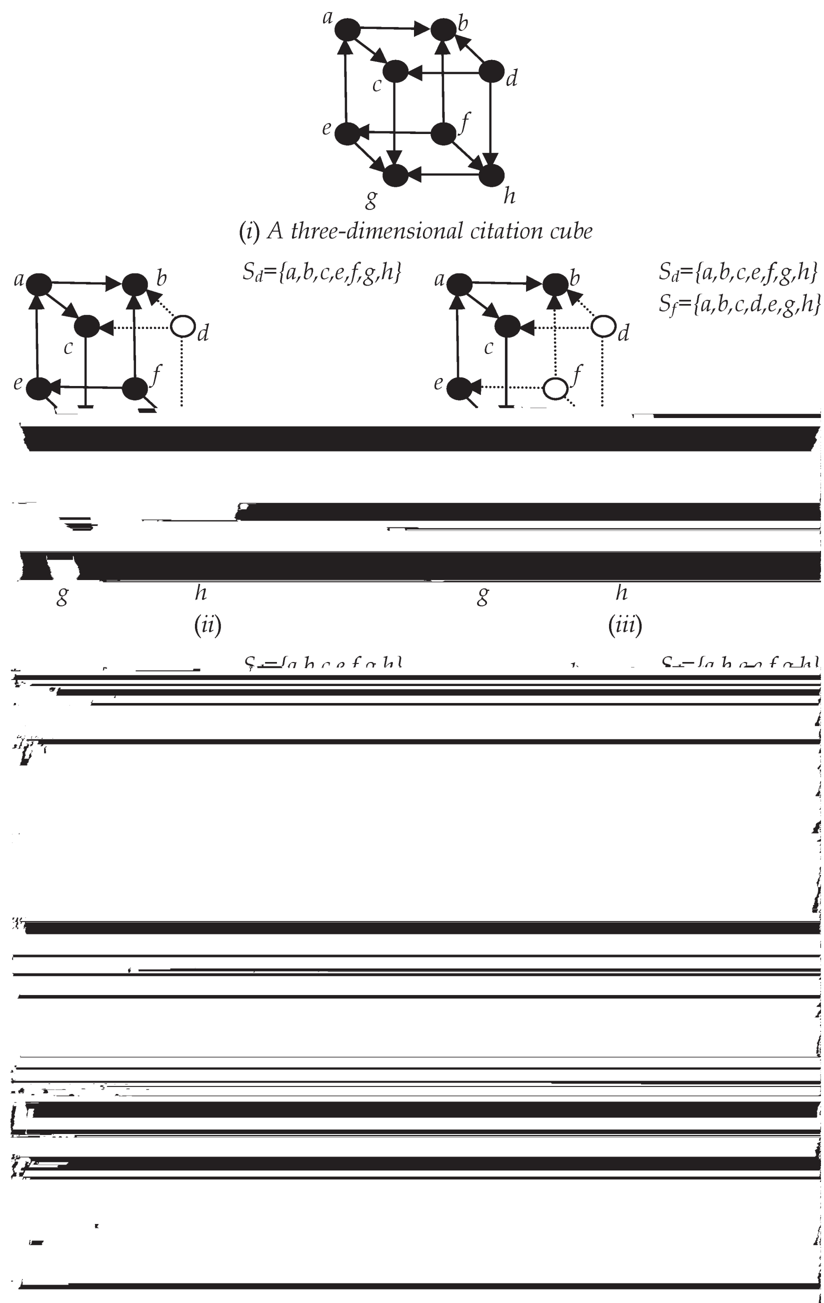


Fig. 9. Case demonstration for the decision making in citation networks.

Moreover, the strategies of $\{a_i | \forall a_i \in (A - (\{a_v\} \cup \sum \Omega'_{a_v}))\}$ do not need to be changed.

2) Given an environment $\langle A, R \rangle$, $a_u, a_v \in A$, if an existing citation link $\langle a_u, a_v \rangle$ is deleted, then the set of strategies $\forall a_i \in (\{a_v\} \cup \sum \Omega'_{a_v})$ can be changed as follows:

$$\forall a_i \in \left(\{a_v\} \cup \sum \Omega'_{a_v}\right), \quad S'_{a_i} = S_{a_i} \cup \left(\{a_u\} \cup \sum \mho'_{a_u} - \sum \mho'_{a_i}\right) \tag{17}$$

where $\sum \Omega'_{a_v}$, $\sum \mho'_{a_u}$, $\sum \mho'_{a_i}$ are the ones in the new structure, S_{ai} is the set of strategies of a_i in the old structure, and S'_{a_i} is the set of strategies of agent a_i in the new structure.

Moreover, the strategies of $\{a_i | \forall a_i \in (A - (\{a_v\} \cup \sum \Omega'_{a_v}))\}$ need not be changed.

Theorem 6: Given an environment $\langle A, R \rangle$ and a decision SL that is useful for the existing citation structure, the adjustment law can obtain useful decision for the new citation structure.

Proof: The proof can be seen in the Appendix. □

Example 9: We can take the citation network and strategies in Fig. 9 (ix) as an example to demonstrate our adjustment law. Fig. 10 shows the adjustment for interaction relation oscillation.

2) Scalability for the Growth of Citation Structures: The growth of citation networks can be based on the dynamics of interacting links that is motivated by the joining agents to

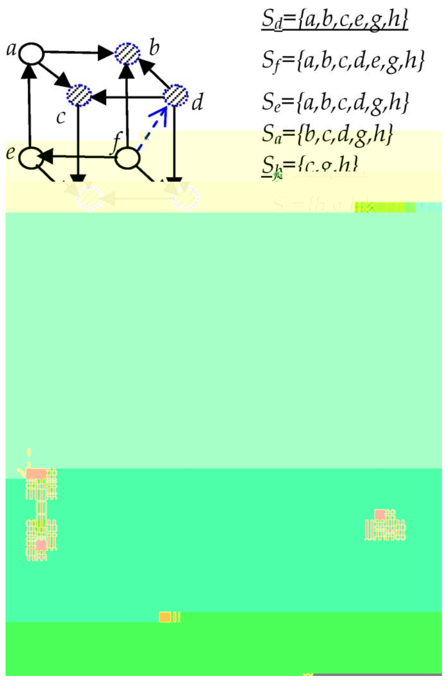


Fig. 10. Case demonstration for the adjustment law: (a) $\langle f, d\rangle$ is added to the citation structure; now, the strategies of agent d, b, c, h, g are adjusted. (b) $\langle d, h\rangle$ is deleted from the citation structure; now, the strategies of agent h and g are adjusted.

construct links (re)directing them toward selected existing agents [21]. If the citation structure at time t is $\langle A_t, R_t\rangle$, where A_t denotes the set of agents and R_t denotes the set of agent citation links, and if the citation structure at time $t+i$ is $\langle A_{t+i}, R_{t+i}\rangle$, where A_{t+i} denotes the set of agents and R_{t+i} denotes the set of agent citation links, then the growth of agent citation structure satisfies the following: $A_t \subseteq A_{t+i}$ and $R_t \subseteq R_{t+i}$. When the citation structure grows, we do not need to make decisions by starting from scratch, which is costly. Thus, we should expand the existing decided strategies locally.

Growth Law: Given an environment $\langle A, R\rangle$ and the existing decision SL, we let an agent a and some citation links associated with it be added to the existing structure, the new set of agents be A', and the new citation structure be R'. The growth of citation links should not produce any cycles in the new citation network; now, we change the strategies of agents according to the following laws.

1) $\forall a_i \in \sum \mho_a$, their strategies are changed as $S'_{a_i} = S_{a_i} \cup \{a\}$.
2) $\forall a_i \in \sum \Omega_a$, their strategies are changed as $S'_{a_i} = S_{a_i} - \{a\} = S_{a_i}$.
3) $\forall a_i \in (A' - \sum \Omega_a - \sum \mho_a - \{a\}) \Rightarrow a_i \not\succ a$, their strategies are changed as $S'_{a_i} = S_{a_i} \cup \{a\}$.
4) For agent a, $S'_a = A' - \{a\} - \sum \mho_a$.

Theorem 7: Obviously, the four parts of the growing law are all determined according to (14) and (15); thus, the growth law can obtain useful decision results.

Example 10: We now take Fig. 9 (ix) as an example; let a new agent z and two new citation links $\langle z, g\rangle$ and $\langle h, z\rangle$ be added to the structure. Now, we can change the decided strategies of the system according to our growth law; the result

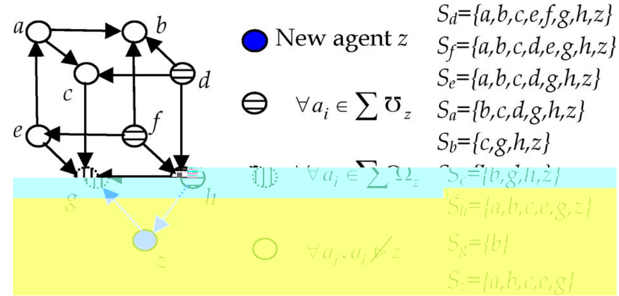


Fig. 11. Case demonstration for the growth law.

is shown in Fig. 11. Obviously, the final decided strategies can satisfy the citation rules for the new citation structure; hence, the decision is useful.

V. Related Work

Our research is related to the decision making of multiagents, where each agent should make decisions about which action to perform to ensure a good joint action for the whole multiagent group. Generally, related work can be categorized as follows.

1) Decision Making of Multiagents Based on Game Theory and Economics [22]: While agents inhabit a shared environment, they negotiate with each other to decide their actions [30]–[34]. To conduct negotiations, they always adopt game theory or other economics techniques, such as bargaining, auction, contracting, etc. The negotiation protocols and decision-making procedures are always focused. The related works include two aspects: cooperative agents and self-interested agents.

In the decision making of cooperative agents, the agents need to cooperate with each other to solve a problem or to reach a common goal. For example, Moehlman *et al.* [23] use decentralized negotiation to solve the distributed planning problem; Lander and Lesser [24] employ multistage negotiation as a means to conduct distributed searches among agents; Pelta and Yager [25] consider a problem of mediated group decision making where a number of agents provide a preference function over a set of alternatives and present an optimization approach for the decision strategies in mediated multiagent negotiations. Another typical example for the decision making of cooperative multiagents is the one in robot soccer, where the agents share a common decision-making criterion and take into account what their partners are able to do [26]. Therefore, in cooperative agents, they always negotiate to reach an agreement, and the decision is made according to the maximum utility of the system.

In the decision making of self-interested agents, the agents try to maximize payoff without concern of the global good; thus, such a self-interested agent will choose the best negotiation strategy for itself [27]. Game theory is a branch of economics that is always used to study interactions between self-interested agents [3]. Game theory may be used to analyze the problems of how interaction strategies can be designed to maximize the welfare of an agent in a multiagent encounter and how protocols or mechanisms can be designed that have certain desirable properties [2]. An agent's el3ers 6trategy bedepend-303.1(ma)-312.4(t)-

information that it has about the preferences and behaviors of other agents. The decision making of self-interested agents is typically seen in the market or electronic commerce [28]. For example, Lomuscio *et al.* [29] present a classification scheme for the negotiation of self-interested agents in electronic commerce.

2) Decision Making of Networked Agents: With large-scale and networked application environments, distributed decision making for the coordination of networked agents has received much attention in recent years. In the related works on decision making of networked agents, a network of agents with initially different opinions can reach a collective decision and hence take action in a distributed manner [11]. Saber and Murray provide convergence, performance, and robustness analyses of an agreement protocol for a network of integrator agents with directed information flow and (perhaps) switching topology, which mainly relies on the tools of algebraic graph theory and matrix theory [10]. Roy *et al.* [11] introduce a quasi-linear stochastic distributed protocol that can be used by a network of sensing agents to reach a collective action agreement; moreover, they put forth the viewpoint that it is useful to consider the information-fusion and decision-making tasks of networks with sensing agents jointly, as a decentralized stabilization or agreement problem. Gal *et al.* [34] describe several new decision-making models that represent, learn, and adapt to various social attributes that influence people's decision making in open mixed networks including agents and people.

3) Modeling the Interdependence Among Multiagents: The dependence among multiagents can be modeled by dependence networks. The dependence network can be used for the study of emerging social structures, such as groups and collectives, and may form a knowledge base for managing complexity in both competitive and organizational or other cooperative contexts [12]. Sichman and Conte [12] model multiagent interdependences among different agents' goals and actions and construct a tool for predicting and simulating their emergence. Wong and Butz [13] propose an automated process for constructing the combined dependence structure of a multiagent probabilistic network, where the dependence structure is a graphical representation of the conditional independencies that are known to hold in the problem domain. Generally, the related works on the interdependence among multiagents mainly focus on the knowledge representation and reasoning dependence among multiagents.

Summarization: The main concerns of related works can be summarized as follows: 1) In the previous decision-making works of multiagents based on game theory and economics, they mainly focus on negotiation protocols and decision-making procedures; 2) in the previous works on the decision making of networked agents, they mainly concern the agreement problems in which all agents in the network must achieve the same opinion and on the connection between the network topology and the performance of the agreement protocol; and 3) in the previous works on the interdependence among multiagents, they mainly focus on the knowledge representation and reasoning dependence among multiagents.

Therefore, previous works seldom take into account the interaction structures of agents. Aiming to solve the structured interaction collision problem of networked multiagents, this paper investigates the interaction structure-oriented decision making.

VI. Conclusion

Networked structures are very popular in the large-scale multiagent systems. We have presented a novel interaction-structure-oriented decision model of networked multiagent strategies, which can satisfy the requirement of interaction structure among agents. The presented model can restrict the strategies of all agents to match their interaction positions. The presented decision-making model contains two parts: One is the basic model for the directed acyclic interaction structure and the other is the extended model for the directed interaction structure with cycles. We theoretically prove that the former can produce the unique outcome, which is to minimize the conflicts among agents, and that the latter can produce the maximum utility. Moreover, the model can produce the same resulted strategies as for isomorphic structures.

Finally, we adopted a multiagent citation network to make a case study. Through the case study, we can then see that our model can minimize collisions for citation relations. In our case study, citation networks are considered DAGs. However, there are also some other special cases of citation structures that are not DAGs, such as mutually citation companion agents and cyclic citation structures occurred in some agents; therefore, we will solve the strategy decision in cyclic citation structures by using our extended model. Moreover, in the future, we will focus on the application of our decision-making model in more complex interaction structures, such as hypergraph, complex social networks, etc.

Appendix
Proof of Theorem 6

Obviously, the adjustment law can result in a useful decision only if we can prove that the adjustment law satisfies the requirements of the decision laws in Section IV-B.

1) Law 1: $\forall a \in A, \sum \Omega_a \subseteq S_a$.

a) $\forall a_i \in (A - \{a_u, a_v\} - \sum \Omega'_{a_v})$

i) $\forall a_j \in (A - \{a_u, a_v\} - \sum \Omega'_{a_v})$, if $a_j \in \sum \Omega_{ai}$, then $a_j \in \sum \Omega'_{a_i}$ and $a_j \in S_{a_i}$. Now, $S'_{a_i} = S_{a_i}$; hence, $a_j \in S'_{a_i}$.

ii) For a_u, if $a_u \in \sum \Omega_{a_i}$, then $a_u \in \sum \Omega'_{a_i}$ and $a_u \in S_{a_i}$. Now, $S'_{a_i} = S_{a_i}$; thus, $a_u \in S'_{a_i}$.

iii) For$\forall a_j \in (\{a_v\} \cup \sum \Omega'_{a_v})$, if $a_j \in \sum \Omega_{a_i}$, then $a_j \in S_{a_i}$.

- When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following occurs.

  $a_j \in \Omega'_{a_i}$ and $S'_{a_i} = S_{a_i}$; thus, we have $a_j \in S'_{a_i}$.

- When an old citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following occurs.

  If $a_j \in \sum \Omega'_{a_i}$, $S'_{a_i} = S_{a_i}$; therefore, we have $a_j \in S'_{a_i}$. If $a_j \notin \sum \Omega'_{a_i}$, then $a_j \not\succ a_i$; therefore, we also have $a_j \in S'_{a_i}$.

b) For agent a_u

i) $\forall a_j \in (A - \{a_u, a_v$

ii) For agent a_u

- When a new citation link $\langle a_u, a_v \rangle$ is added to the citation structure, the following happens.

  Obviously, $a_i \not\triangleright a_j$ is not true in the new structures; hence, we need not address them.

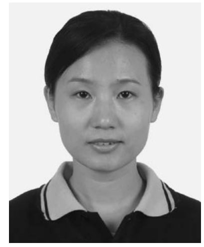
- When an existing citation link $\langle a_u, a_v \rangle$ is deleted from the citation structure, the following occurs.

  If $a_i \not\triangleright a_u$ or $a_i \in \sum \Omega_{a_u}$ is in the old citation structure, then we have $a_i \in S_{a_u}$. We have $S'_{a_u} = S_{a_u} \Rightarrow a_i \in S'_{a_u}$

Yichuan Jiang (M'07) received the Ph.D. degree in computer science from Fudan University, Shanghai, China, in 2005.

From July to October 2005, he was a Postdoctoral Researcher with the Department of Computer Science, Hong Kong Baptist University, Kowloon, Hong Kong. He was then a Japan Society for the Promotion of Science Research Fellow with the Department of Social Informatics, Kyoto University, Kyoto, Japan. He is currently a Professor with the Laboratory of Complex Systems and Social Computing, School of Computer Science and Engineering, Southeast University, Nanjing, China. His current research interests include multiagent systems, complex distributed systems, social networks, and social computing. He has authored or coauthored more than 60 scientific articles published in refereed journals and conferences, such as the IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS, the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART C: APPLICATIONS AND REVIEWS, the *Journal of Parallel and Distributed Computing*, the International Joint Conference on Artificial Intelligence, and the International Conference on Autonomous Agents and Multiagent Systems. He is also a member of editorial board of *Advances in the Internet of Things* and of the *Chinese Journal of Computers*.

Dr. Jiang is a member of the IEEE Computer Society and the Association for Computing Machinery and a senior member of China Computer Federation and The Chinese Institute of Electronics. He was awarded the New Century Excellent Talents in University of State Education Ministry of China award, the Best Paper Award from PRIMA2006, the Nomination Award for the National Excellent Doctoral Dissertation of China, and the Young Distinguished Professor award from Southeast University. He is also the recipient of the HwaYing Young Scholar Award in 2008 from The HwaYing Education and Culture Foundation.



Jing Hu received the Ph.D. degree from Nanjing University, Nanjing, China, in 2005.

She is currently an Associate Professor with the College of Arts, Nanjing University of Aeronautics and Astronautics, Nanjing. Her current research interests include multiagent-based virtual drama, multiagent modeling for arts, and mind-art computation.

Donghui Lin received the Ph.D. degree in informatics from Kyoto University, Kyoto, Japan, in 2008.

Since then, he has been a Researcher with the Knowledge Creating Communication Research Center, National Institute of Information and Communications Technology, Tokyo, Japan. He has authored or coauthored more than 20 scientific articles published in refereed journals and conferences. His current research interests include multiagent systems and services computing.